Favourite generations and range of a randomly biased random walk on Galton-Watson tree

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Summary. Suppose that we have a supercritical Galton-Watson tree $\mathbb{T}$ rooted at ϕ, with offspring ν. We can define a random walk $(X_i; i \geq 0)$ on $\mathbb{T}$ started from $X_0 = \phi$, by choosing uniformly one vertex among the neighbours of X_i at the $(i+1)$-th step. And the biased random walk is define as follows. Take a constant $\lambda > 0$ which serves as the bias. The probabilities of transition are

$$p(u,v) = \begin{cases} \frac{\lambda}{1+\sum_{z:\text{ child of u}} \lambda} & \text{if v is child of u;} \\ \frac{1}{1+\sum_{z:\text{ child of u}} \lambda} & \text{if v is parent of u.} \end{cases}$$

More generally, we attach to each vertex $u \in \mathbb{T}$ an i.i.d. positive random variable A_u and run a stochastically biased random walk $(X_i; i \geq 0)$ started from the root so that its transition probabilities are

$$p(u,v) = \begin{cases} \frac{A_v}{1+\sum_{z:\text{ child of u}} A_z} & \text{if v is child of u;} \\ \frac{1}{1+\sum_{z:\text{ child of u}} A_z} & \text{if v is parent of u.} \end{cases}$$

We are interested in the boundary case where $\mathbf{E}[\sum_{i=1}^{\nu} A_i] = 1$ and $\mathbf{E}[\sum_{i=1}^{\nu} A_i \log A_i] = 0$. Hu and Shi [2] proved that the generation of X_n is of order $(\log n)^2$ in distribution. We count, up to time n, the visited sites in one generation $\gamma(\log n)^2$ with $\gamma > 0$ some fixed constant, and find out that the number is approximately $\lambda(\gamma)\frac{n}{(\log n)^3}$ in probability. From this result, we prove that the range of the random walk is of order $\frac{n}{\log n}$ up to time n.

Keywords. Biased random walk; branching random walk.

References

[1] Andreoletti, P. and Chen, X. (2015) Range and critical generations of a randomly biased random walk on Galton-Watson tree. (In preparation)

[2] Hu, Y. and Shi, Z. (2015) The slow regime of randomly biased walks on trees. `arXiv:1501.07700`.