Favourite generations and range of a randomly biased random walk on Galton-Watson tree

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**Summary.** Suppose that we have a supercritical Galton-Watson tree $T$ rooted at $\phi$, with offspring $\nu$. We can define a random walk $(X_i; i \geq 0)$ on $T$ started from $X_0 = \phi$, by choosing uniformly one vertex among the neighbours of $X_i$ at the $(i + 1)$-th step. And the biased random walk is define as follows. Take a constant $\lambda > 0$ which serves as the bias. The probabilities of transition are

$$p(u, v) = \begin{cases} \frac{\lambda}{1 + \sum_{z: \text{child of } u} \lambda} & \text{if } v \text{ is child of } u; \\
\frac{1}{1 + \sum_{z: \text{child of } u} \lambda} & \text{if } v \text{ is parent of } u. \end{cases}$$

More generally, we attach to each vertex $u \in T$ an i.i.d. positive random variable $A_u$ and run a stochastically biased random walk $(X_i; i \geq 0)$ started from the root so that its transition probabilities are

$$p(u, v) = \begin{cases} \frac{A_v}{1 + \sum_{z: \text{child of } u} A_z} & \text{if } v \text{ is child of } u; \\
\frac{1}{1 + \sum_{z: \text{child of } u} A_z} & \text{if } v \text{ is parent of } u. \end{cases}$$

We are interested in the boundary case where $E[\sum_{i=1}^{\nu} A_i] = 1$ and $E[\sum_{i=1}^{\nu} A_i \log A_i] = 0$. Hu and Shi [2] proved that the generation of $X_n$ is of order $(\log n)^2$ in distribution. We count, up to time $n$, the visited sites in one generation $\gamma (\log n)^2$ with $\gamma > 0$ some fixed constant, and find out that the number is approximately $\lambda (\gamma) \frac{n}{(\log n)^2}$ in probability. From this result, we prove that the range of the random walk is of order $\frac{n}{\log n}$ up to time $n$.

**Keywords.** Biased random walk; branching random walk.

**References**
