1 Procédures générales

```
> # Uq(D21) programe Maple pour calcul du fonteur ribbon sur des
> # diagrammes élémentaires coloriés par l'analogue quantique de
la
> # représentation adjointe.
> #
> # Les vecteurs de bases sont numérotés de 1 à 17.  Chacun a un
poids
> # appartenant à l'ensemble des racines.  Un tenseur de L@n
> # est représenté par une liste de monomes.  Chaque monome est
un couple
> # formé d'un coefficient et de de la liste des vecteurs qui le
> # compose.
> #
> racine:=[[0,0,0], [0,0,0], [0,0,0], [0, 0, 2], [1, -1, 1], [1,
1, 1],
> [2, 0, 0], [1, -1, -1], [1, 1, -1], [0, 2, 0], [0, 0, -2], [-1,1,-1],
> [-1, -1, -1], [-2, 0, 0], [-1, 1, 1], [-1, -1, 1], [0, -2, 0]];
> racine_set:={[0, 0, 2], [1, -1, 1], [1, 1, 1], [2, 0, 0], [1,
-1,
> -1],
> [1, 1, -1], [0, 2, 0], [0, 0, -2], [-1,1,-1], [-1, -1, -1], [-2,
0,
> 0], [-1, 1, 1], [-1, -1, 1], [0, -2, 0]};
> racine_set0:={[0,0,0], [0, 0, 2], [1, -1, 1], [1, 1, 1], [2,
0, 0],
> [1, -1, -1], [1, 1, -1], [0, 2, 0], [0, 0, -2], [-1,1,-1], [-1,
-1,
> -1], [-2, 0, 0], [-1, 1, 1], [-1, -1, 1], [0, -2, 0]};
> for i from 4 to 17 do vec[(racine[i])]:=i od:  eval(vec):
> parite:=[0,0,0,0,1,1,0,1,1,0,0,1,1,0,1,1,0];
> MJ:=matrix(17,17,0):  for i to 17 do MJ[i,i]:=1-2*parite[i] od:
> for i to 3 do q[i]:=a[i]^2 od:
> a[1]:=1/a[2]/a[3];
> #(***************************************************************
> *)
> #(*****************Procédures
> générales********************************)
> #(***************************************************************
> *)
> listorder:=proc(l,ll)
> local i,n;
> if l=ll then false else
> n:=nops(l);
> if n>nops(ll) then not(listorder(ll,l)) else
> for i to n while op(i,l)=op(i,ll) do od;
> if i=n+1 then true else evalb(op(i,l)<op(i,ll));
> fi; fi; fi; end;
> sndlex:=proc(l,ll) listorder(op(2,l),op(2,ll)); end;
> scal:=proc(k,l) map(proc(m,k) [normal(k*op(1,m)),op(2,m)] end,l,k)
> end;
> vide:=proc(x) local k; k:=normal(op(1,x));
> if k<>0 then [k,op(2,x)] else NULL;
> fi end;
> simp:=proc(ll)
> local i,l,x;
> l:=array(1..nops(ll),sort(ll,sndlex));
> for i to nops(ll)-1 do
> x:=l[i+1];
> if op(2,l[i])=op(2,x) then
> l[i+1]:=[op(1,l[i])+op(1,x),op(2,x)]; l[i]:=[0,[]]
> fi od;
> map(vide,convert(l,list));
> end;
> cl:=proc(ca,cb,la,lb) simp([op(scal(ca,la)),op(scal(cb,lb))])
end;
> EqM:=proc(M) convert(map(op,convert(evalm(M),listlist)),set)
end;
> EqL:=proc(t) convert(map(y->y[1],t),set) end;
> ev:=proc(y) map(factor@eval,evalm(y)) end;
```

$racine := [[0, 0, 0], [0, 0, 0], [0, 0, 0], [0, 0, 2], [1, -1, 1], [1, 1, 1], [2, 0, 0], [1, -1, -1],$
$[1, 1, -1], [0, 2, 0], [0, 0, -2], [-1, 1, -1], [-1, -1, -1], [-2, 0, 0], [-1, 1, 1], [-1, -1, 1],$
$[0, -2, 0]]$

$racine\_set := \{[1, -1, 1], [1, 1, 1], [0, 0, 2], [0, 0, -2], [-1, -1, -1], [-2, 0, 0], [-1, 1, -1],$
$[2, 0, 0], [1, 1, -1], [0, 2, 0], [1, -1, -1], [-1, 1, 1], [0, -2, 0], [-1, -1, 1]\}$

$racine\_set0 := \{[1, -1, 1], [1, 1, 1], [0, 0, 2], [0, 0, 0], [0, 0, -2], [-1, -1, -1], [-2, 0, 0],$
$[-1, 1, -1], [2, 0, 0], [1, 1, -1], [0, 2, 0], [1, -1, -1], [-1, 1, 1], [0, -2, 0], [-1, -1, 1]\}$

$$parite := [0, 0, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0]$$

$$a_1 := \frac{1}{a_2\, a_3}$$

```
listorder := proc(l, ll)
local i, n;
    if l = ll then false
    else
        n := nops(l) ;
        if nops(ll) < n then  not listorder(ll, l)
        else
            for i to n while  op(i, l) = op(i, ll) do  end do ;
            if i = n + 1 then true else evalb(op(i, l) < op(i, ll)) end if
        end if
    end if
end proc
```

```
sndlex := proc(l, ll) listorder(op(2, l), op(2, ll)) end proc
```

```
scal := proc(k, l)
    map(proc(m, k) [normal(k * op(1, m)), op(2, m)] end proc, l, k)
end proc
```

```
vide := proc(x)
local k;
    k := normal(op(1, x)) ; if k ≠ 0 then [k, op(2, x)] else NULL end if
end proc
```

$$
\begin{array}{l}
simp := \mathbf{proc}(ll)\\
\mathbf{local}\, i,\, l,\, x;\\
\quad l := \mathrm{array}(1..\mathrm{nops}(ll),\, \mathrm{sort}(ll,\, sndlex))\,;\\
\quad \mathbf{for}\, i\, \mathbf{to}\, \mathrm{nops}(ll) - 1\, \mathbf{do}\\
\quad\quad x := l_{i+1}\,;\\
\quad\quad \mathbf{if}\, \mathrm{op}(2,\, l_i) = \mathrm{op}(2,\, x)\, \mathbf{then}\\
\quad\quad\quad l_{i+1} := [\mathrm{op}(1,\, l_i) + \mathrm{op}(1,\, x),\, \mathrm{op}(2,\, x)]\,;\, l_i := [0,\, []]\\
\quad\quad \mathbf{end\ if}\\
\quad \mathbf{end\ do};\\
\quad \mathrm{map}(vide,\, \mathrm{convert}(l,\, list))\\
\mathbf{end\ proc}
\end{array}
$$

$$cl := \mathbf{proc}(ca,\, cb,\, la,\, lb)\, \mathrm{simp}([\mathrm{op}(\mathrm{scal}(ca,\, la)),\, \mathrm{op}(\mathrm{scal}(cb,\, lb))])\, \mathbf{end\ proc}$$

$$EqM := \mathbf{proc}(M)\, \mathrm{convert}(\mathrm{map}(op,\, \mathrm{convert}(\mathrm{evalm}(M),\, listlist)),\, set)\, \mathbf{end\ proc}$$

$$EqL := \mathbf{proc}(t)\, \mathrm{convert}(\mathrm{map}(y \to y_1,\, t),\, set)\, \mathbf{end\ proc}$$

$$ev := \mathbf{proc}(y)\, \mathrm{map}(factor@eval,\, \mathrm{evalm}(y))\, \mathbf{end\ proc}$$

$$ttrace := \mathbf{proc}(fname)\, \ldots\, \mathbf{end\ proc}$$

```
Warning, the protected names norm and trace have been redefined and
unprotected
```

$$cr := \mathbf{proc}(A,\, B)\, `\&*`(A,\, B) - `\&*`(B,\, A)\, \mathbf{end\ proc}$$

$$qn := \mathbf{proc}(i,\, j)\, {a_i}^j - 1/{a_i}^j\, \mathbf{end\ proc}$$

2 Matrices de la représentation M

```
> #(******************************************************************
> )
> #(******************Matrices de la représentation M******************)
> #(*****************************************************************)
> for i to 3 do M[i]:=linalg[diag](seq(a[i]^racine[j][i],j=1..17));
> M[-i]:=linalg[inverse](M[i]) od;
> n:=8; M[n]:=matrix(17,17): assign({M[8][17,16] = p1*Qn[2,2]*a[2]^2,
> M[8][1,15] = 1/gamma*p1, M[8][7,6] = p1*Qn[1,2], M[8][5,4] =
> -Qn[3,2]*p1, M[8][8,2] = -p1*Qn[2,2], M[8][8,3] = -Qn[3,2]*p1,
> M[8][11,12] = Qn[3,2]*a[3]^2*p1, M[8][9,10] = -p1*Qn[2,2], M[8][13,14]
> = p1*Qn[1,2]*a[2]^2*a[3]^2}): for i to 17 do for j to 17 do
if
> M[n][i,j]=evaln(M[n][i,j]) then M[n][i,j]:=0 fi od od:
> n:=15; M[n]:=matrix(17,17): assign({M[15][1,8] = phi/gamma,
> M[15][4,5]
> = 1, M[15][16,17] = 1/(a[2]^2), M[15][15,3] = Qn[3,2]*phi,
> M[15][12,11] = 1/(a[3]^2), M[15][10,9] = 1, M[15][6,7] = 1,
> M[15][14,13] = -1/(a[2]^2*a[3]^2), M[15][15,2] = Qn[2,2]*phi}):
for i
> to 17 do for j to 17 do if M[n][i,j]=evaln(M[n][i,j]) then
> M[n][i,j]:=0 fi od od:
> n:=4; M[n]:=matrix(17,17): assign({M[4][5,8] = 1, M[4][15,12]
= -1,
> M[4][16,13] = -1, M[4][4,3] = Qn[3,4]/Qn[3,2], M[4][3,11] =
> -1/(phi*Qn[3,2]*a[3]^2), M[4][4,1] = gamma/phi, M[4][6,9] = 1}):
for
> i
> to 17 do for j to 17 do if M[n][i,j]=evaln(M[n][i,j]) then
> M[n][i,j]:=0 fi od od:
> n:=11; M[n]:=matrix(17,17): assign({M[11][11,1] =
> -Qn[3,2]*a[3]^2*gamma, M[11][13,16] = -1, M[11][8,5] = 1, M[11][12,15]
> = -1, M[11][9,6] = 1, M[11][11,3] = -a[3]^2*Qn[3,4]*phi, M[11][3,4]
=
> 1} ): for i to 17 do for j to 17 do if M[n][i,j]=evaln(M[n][i,j])
> then
> M[n][i,j]:=0 fi od od:
> n:=10; M[n]:=matrix(17,17): assign({M[10][15,16] = -1, M[10][10,1]
=
> gamma/phi, M[10][10,2] = Qn[2,4]/Qn[2,2], M[10][12,13] = -1,
> M[10][2,17] = -1/(phi*Qn[2,2]*a[2]^2), M[10][6,5] = 1, M[10][9,8]
=
> 1}): for i to 17 do for j to 17 do if M[n][i,j]=evaln(M[n][i,j])
then
> M[n][i,j]:=0 fi od od:
> n:=17; M[n]:=matrix(17,17): assign({M[17][2,10] = 1, M[17][17,1]
=
> -gamma*Qn[2,2]*a[2]^2, M[17][17,2] = -a[2]^2*Qn[2,4]*phi, M[17][16,15]
> = -1, M[17][13,12] = -1, M[17][5,6] = 1, M[17][8,9] = 1}): for
i to
> 17
> do for j to 17 do if M[n][i,j]=evaln(M[n][i,j]) then M[n][i,j]:=0
fi
> od od:
> M[5]:=ev(M[8]&*M[4]-1/a[3]^2*M[4]&*M[8]):
> M[9]:=ev(M[10]&*M[8]-1/a[2]^2*M[8]&*M[10]):
> M[6]:=ev(M[10]&*M[5]-1/a[2]^2*M[5]&*M[10]):
> M[7]:=ev(M[8]&*M[6]+a[1]^2*M[6]&*M[8]):
> M[12]:=ev(M[15]&*M[11]-1/a[3]^2*M[11]&*M[15]):
> M[16]:=ev(M[17]&*M[15]-1/a[2]^2*M[15]&*M[17]):
> M[13]:=ev(M[17]&*M[12]-1/a[2]^2*M[12]&*M[17]):
> M[14]:=ev(M[15]&*M[13]+a[1]^2*M[13]&*M[15]):
> for i to 17 do tM[i]:=ev(linalg[transpose](M[i])) od:
> rel:=map(factor,[
> EqM(M[8]&*M[8]),
> EqM(M[15]&*M[15]),
> EqM(cr(M[4],M[17])),
> EqM(cr(M[4],M[15])),
> EqM(cr(M[11],M[10])),
> EqM(cr(M[11],M[8])),
```

$$M_1 := \begin{bmatrix}
1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, \frac{1}{{a_2}^2\, {a_3}^2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, a_2\, a_3, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, a_2\, a_3, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, {a_2}^2\, {a_3}^2, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, a_2\, a_3, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, a_2\, a_3, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1
\end{bmatrix}$$

$$M_{-1} := \begin{bmatrix}
1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, a_2\, a_3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, a_2\, a_3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, {a_2}^2\, {a_3}^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, a_2\, a_3, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, a_2\, a_3, 0, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{{a_2}^2\, {a_3}^2}, 0, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1}{a_2\, a_3}, 0 \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1
\end{bmatrix}$$

$$M_2 := \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & {a_2}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{{a_2}^2}
\end{bmatrix}$$

$$M_{-2} := \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{{a_2}^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & {a_2}^2
\end{bmatrix}$$

$$
M_3 := \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & {a_3}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{{a_3}^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
M_{-3} := \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{{a_3}^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & {a_3}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a_3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$n := 8$$
$$n := 15$$
$$n := 4$$
$$n := 11$$
$$n := 10$$
$$n := 17$$
$$rel := [\{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}, \{0\}]$$

3 Calcul de la R-matrice

```
> #(**************************************************************
> )
> #(********************* Calcul de la R-matrice********************)
> #(**************************************************************)
> #poids de L@L
> weight:=NULL:
> for i to 17 do
> for j to 17 do
> weight:=weight,convert(ev(racine[i]+racine[j]),list) od od:
> weight:={weight}; nops(weight);
> #terme de contribution non nulle dans M(R)
> Rterme:=NULL:
> for i1 from 0 to 2 do
> for i2 from 0 to 1 do
> for i3 from 0 to 1 do
> for i4 from 0 to 2 do
> for i5 from 0 to 1 do
> for i6 from 0 to 1 do
> for i7 from 0 to 2 do
> if
> member(convert(ev(i1*racine[4]+i2*racine[5]
> +i3*racine[6]+i4*racine[7]
> +i5*racine[8]+i6*racine[9]
> +i7*racine[10]),list),weight)
> then Rterme:=Rterme,[i1,i2,i3,i4,i5,i6,i7]
> fi
> od od od od od od od:
> Rterme:=[Rterme]; nRterme:=nops(Rterme);
> f:=proc(l) local ll,i,j; ll:=NULL;
> for i to 7 do for j to l[i] do ll:=ll,i+3 od od:  [ll] end;
> Rterm:= map(f,Rterme);
> #Rterm:liste des monomes en les vecteurs de racines de UqD21
> contribuant
> fw:=proc(l) local w,i; w:=[0,0,0];
> for i to 7 do w:=ev(w+l[i]*racine[i+3]) od:
> convert(w,list) end;
> Rw:=map(fw,Rterme);
> #Rw:  R= sum X@Y avec weight(X)=-weight(Y)=Rw[i]
> Rcontrib:=proc(i,j) local k,l; l:=NULL;
> for k to nRterme do
> if member(convert(ev(racine[i]+Rw[k]),list),racine_set0)
> and member(convert(ev(racine[j]-Rw[k]),list),racine_set0)
> then l:=l,k fi od:  [l] end;
> #renvoie les monomes de Rterm contribuant à R(v_i@v_j)
> epsilon:=p1*a[2]^2*a[3]^2/((a[2]^4+1)*(a[3]^4+1));
> Rcoef:=table([
> (1, 6) = -a[2]^4*a[3]^2/(epsilon*(a[3]^4+1)*(a[2]^4+1)),
> (1, 1) = -(a[3]^2+1)*(a[3]+1)*(a[3]-1)/(a[3]^2),
> (1, 7) = -(a[2]^2+1)*(a[2]+1)*(a[2]-1)/(a[2]^2),
> (1, 2) = -a[3]^4*a[2]^2/((a[3]^4+1)*(a[2]^4+1)*epsilon),
> (2, 1) = (a[3]-1)^2*(a[3]+1)^2*(a[3]^2+1)^2/(a[3]^4*(a[3]^4+1)),
> (1, 3) = a[3]^4*a[2]^4/((a[3]^4+1)*(a[2]^4+1)*epsilon),
> (1, 4) = -a[3]^10*a[2]^10/((a[3]^4+1)^2*(a[2]^4+1)^2*epsilon^2*
> (a[2]^2*a[3]^2+1)*(a[2]*a[3]+1)
> *(a[2]*a[3]-1)),
> (2, 7) = (a[2]^2+1)^2*(a[2]+1)^2*(a[2]-1)^2/(a[2]^4*(a[2]^4+1)),
> (1, 5) = a[2]^2*a[3]^2/(epsilon*(a[3]^4+1)*(a[2]^4+1)),
> (2, 4) = a[2]^24*a[3]^24/(epsilon^4*(a[3]^4+1)^4*(a[2]^4+1)^4*
> (a[2]^4*a[3]^4+1)*(a[2]^2*a[3]^2+1)^2
> *(a[2]*a[3]+1)^2*(a[2]*a[3]-1)^2)]);
> Coefmo:=proc(t) local i,r,c; c:=1; r:=Rterme[t];
> for i to 7 do if r[i]>0 then c:=c*Rcoef[r[i],i] fi od:
> (-1)^(r[2]*r[3]+r[2]*r[5]+r[2]*r[6]+r[3]*r[5]+r[3]*r[6]+r[5]*r[6])*c
> end;
> # coeffs des monomes dans l'expression de la R-matrice universelle
> f:=proc(l) (-1)^(l[2]+l[6]+l[3]+l[5]) end;
> Rtpar:=map(f,Rterme):
> #Rtpar:  R= sum X@Y avec parité(X)=parité(Y)=Rtpar[i]
> Rmm:=proc(i,j) local ri, rj, s, eij, rc, t, mo, p, k;
> ri:=racine[i]; rj:=racine[j]; s:=matrix(17,17,0);
```

$\mathit{weight} := \{[1, -3, -1], [0, -2, 2], [-1, 1, 3], [1, -1, 1], [1, 1, 1], [0, 0, 2], [0, 0, 0], [0, 0, -2],$
$[-1, -1, -1], [-2, 0, 0], [-1, 1, -1], [2, 0, 0], [1, 1, -1], [0, 2, 0], [1, -1, -1], [-1, 1, 1],$
$[0, -2, 0], [-1, -1, 1], [0, 2, -2], [0, -2, -2], [1, -1, -3], [-2, 0, 2], [-1, 3, 1], [-1, -1, 3],$
$[-1, -3, 1], [1, 1, -3], [1, -3, 1], [2, -2, -2], [3, -1, 1], [3, 1, 1], [-2, 0, -2], [-1, 1, -3],$
$[-1, 3, -1], [2, 2, 0], [-1, -3, -1], [3, -1, -1], [0, 4, 0], [2, -2, 0], [-1, -1, -3], [-2, 2, 0],$
$[0, 0, -4], [-3, 1, 1], [4, 0, 0], [0, 0, 4], [2, 2, 2], [-2, -2, -2], [-2, 2, -2], [2, -2, 2],$
$[3, 1, -1], [-3, -1, 1], [1, -1, 3], [1, 1, 3], [2, 0, 2], [-3, -1, -1], [-4, 0, 0], [-3, 1, -1],$
$[-2, -2, 2], [2, 0, -2], [0, 2, 2], [2, 2, -2], [-2, 2, 2], [-2, -2, 0], [1, 3, 1], [1, 3, -1],$
$[0, -4, 0]\}$

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$\mathit{Rterme} := [[0, 0, 0, 0, 0, 0, 0], [0, 0, 0, 0, 0, 0, 1], [0, 0, 0, 0, 0, 0, 2], [0, 0, 0, 0, 0, 1, 0],$
$[0, 0, 0, 0, 0, 1, 1], [0, 0, 0, 0, 1, 0, 0], [0, 0, 0, 0, 1, 0, 1], [0, 0, 0, 0, 1, 0, 2],$
$[0, 0, 0, 0, 1, 1, 0], [0, 0, 0, 0, 1, 1, 1], [0, 0, 0, 1, 0, 0, 0], [0, 0, 0, 1, 0, 0, 1],$
$[0, 0, 0, 1, 0, 1, 0], [0, 0, 0, 1, 1, 0, 0], [0, 0, 0, 1, 1, 0, 1], [0, 0, 0, 2, 0, 0, 0],$
$[0, 0, 1, 0, 0, 0, 0], [0, 0, 1, 0, 0, 0, 1], [0, 0, 1, 0, 0, 1, 0], [0, 0, 1, 0, 1, 0, 0],$
$[0, 0, 1, 0, 1, 0, 1], [0, 0, 1, 0, 1, 1, 0], [0, 0, 1, 1, 0, 0, 0], [0, 0, 1, 1, 1, 0, 0],$
$[0, 1, 0, 0, 0, 0, 0], [0, 1, 0, 0, 0, 0, 1], [0, 1, 0, 0, 0, 0, 2], [0, 1, 0, 0, 0, 1, 0],$
$[0, 1, 0, 0, 0, 1, 1], [0, 1, 0, 0, 1, 0, 0], [0, 1, 0, 0, 1, 0, 1], [0, 1, 0, 0, 1, 0, 2],$
$[0, 1, 0, 0, 1, 1, 0], [0, 1, 0, 0, 1, 1, 1], [0, 1, 0, 1, 0, 0, 0], [0, 1, 0, 1, 0, 0, 1],$
$[0, 1, 0, 1, 0, 1, 0], [0, 1, 0, 1, 1, 0, 1], [0, 1, 1, 0, 0, 0, 0], [0, 1, 1, 0, 0, 0, 1],$
$[0, 1, 1, 0, 0, 1, 0], [0, 1, 1, 0, 1, 0, 0], [0, 1, 1, 0, 1, 0, 1], [0, 1, 1, 0, 1, 1, 0],$
$[1, 0, 0, 0, 0, 0, 0], [1, 0, 0, 0, 0, 0, 1], [1, 0, 0, 0, 0, 1, 0], [1, 0, 0, 0, 0, 1, 1],$
$[1, 0, 0, 0, 1, 0, 0], [1, 0, 0, 0, 1, 0, 1], [1, 0, 0, 0, 1, 0, 2], [1, 0, 0, 0, 1, 1, 0],$
$[1, 0, 0, 0, 1, 1, 1], [1, 0, 0, 1, 0, 0, 0], [1, 0, 0, 1, 0, 0, 1], [1, 0, 0, 1, 0, 1, 0],$
$[1, 0, 0, 1, 1, 0, 0], [1, 0, 0, 1, 1, 0, 1], [1, 0, 0, 1, 1, 1, 0], [1, 0, 1, 0, 0, 0, 0],$
$[1, 0, 1, 0, 0, 1, 0], [1, 0, 1, 0, 1, 0, 0], [1, 0, 1, 0, 1, 0, 1], [1, 0, 1, 0, 1, 1, 0],$
$[1, 1, 0, 0, 0, 0, 0], [1, 1, 0, 0, 0, 0, 1], [1, 1, 0, 0, 0, 1, 0], [1, 1, 0, 0, 0, 1, 1],$
$[1, 1, 0, 0, 1, 0, 0], [1, 1, 0, 0, 1, 0, 1], [1, 1, 0, 0, 1, 0, 2], [1, 1, 0, 0, 1, 1, 0],$
$[1, 1, 0, 0, 1, 1, 1], [2, 0, 0, 0, 0, 0, 0], [2, 0, 0, 0, 0, 1, 0], [2, 0, 0, 0, 1, 0, 0],$
$[2, 0, 0, 0, 1, 0, 1], [2, 0, 0, 0, 1, 1, 0], [2, 0, 0, 0, 1, 1, 1]]$

$$\mathit{nRterme} := 79$$

$f :=$ **proc**(l)
local $ll,\ i,\ j;$
$ll := \mathit{NULL}$; **for** i **to** 7 **do for** j **to** l_i **do** $ll := ll,\ i+3$ **end do end do**; $[ll]$
end proc

$Rterm := [[], [10], [10, 10], [9], [9, 10], [8], [8, 10], [8, 10, 10], [8, 9], [8, 9, 10], [7],$
$[7, 10], [7, 9], [7, 8], [7, 8, 10], [7, 7], [6], [6, 10], [6, 9], [6, 8], [6, 8, 10], [6, 8, 9],$
$[6, 7], [6, 7, 8], [5], [5, 10], [5, 10, 10], [5, 9], [5, 9, 10], [5, 8], [5, 8, 10],$
$[5, 8, 10, 10], [5, 8, 9], [5, 8, 9, 10], [5, 7], [5, 7, 10], [5, 7, 9], [5, 7, 8, 10], [5, 6],$
$[5, 6, 10], [5, 6, 9], [5, 6, 8], [5, 6, 8, 10], [5, 6, 8, 9], [4], [4, 10], [4, 9], [4, 9, 10],$
$[4, 8], [4, 8, 10], [4, 8, 10, 10], [4, 8, 9], [4, 8, 9, 10], [4, 7], [4, 7, 10], [4, 7, 9],$
$[4, 7, 8], [4, 7, 8, 10], [4, 7, 8, 9], [4, 6], [4, 6, 9], [4, 6, 8], [4, 6, 8, 10], [4, 6, 8, 9],$
$[4, 5], [4, 5, 10], [4, 5, 9], [4, 5, 9, 10], [4, 5, 8], [4, 5, 8, 10], [4, 5, 8, 10, 10],$
$[4, 5, 8, 9], [4, 5, 8, 9, 10], [4, 4], [4, 4, 9], [4, 4, 8], [4, 4, 8, 10], [4, 4, 8, 9],$
$[4, 4, 8, 9, 10]]$

$fw :=$ **proc**(l)
local w, i;
$w := [0, 0, 0]$; **for** i **to** 7 **do** $w := \mathrm{ev}(w + l_i * racine_{i+3})$ **end do**; convert(w, *list*)
end proc

$Rw := [[0, 0, 0], [0, 2, 0], [0, 4, 0], [1, 1, -1], [1, 3, -1], [1, -1, -1], [1, 1, -1], [1, 3, -1],$
$[2, 0, -2], [2, 2, -2], [2, 0, 0], [2, 2, 0], [3, 1, -1], [3, -1, -1], [3, 1, -1], [4, 0, 0],$
$[1, 1, 1], [1, 3, 1], [2, 2, 0], [2, 0, 0], [2, 2, 0], [3, 1, -1], [3, 1, 1], [4, 0, 0],$
$[1, -1, 1], [1, 1, 1], [1, 3, 1], [2, 0, 0], [2, 2, 0], [2, -2, 0], [2, 0, 0], [2, 2, 0],$
$[3, -1, -1], [3, 1, -1], [3, -1, 1], [3, 1, 1], [4, 0, 0], [4, 0, 0], [2, 0, 2], [2, 2, 2],$
$[3, 1, 1], [3, -1, 1], [3, 1, 1], [4, 0, 0], [0, 0, 2], [0, 2, 2], [1, 1, 1], [1, 3, 1],$
$[1, -1, 1], [1, 1, 1], [1, 3, 1], [2, 0, 0], [2, 2, 0], [2, 0, 2], [2, 2, 2], [3, 1, 1],$
$[3, -1, 1], [3, 1, 1], [4, 0, 0], [1, 1, 3], [2, 2, 2], [2, 0, 2], [2, 2, 2], [3, 1, 1],$
$[1, -1, 3], [1, 1, 3], [2, 0, 2], [2, 2, 2], [2, -2, 2], [2, 0, 2], [2, 2, 2], [3, -1, 1],$
$[3, 1, 1], [0, 0, 4], [1, 1, 3], [1, -1, 3], [1, 1, 3], [2, 0, 2], [2, 2, 2]]$

$Rcontrib :=$ **proc**(i, j)
local k, l;
$l := NULL$;
for k **to** $nRterme$ **do**
ifmember(convert(ev($racine_i + Rw_k$), *list*), *racine_set0*) **and**
member(convert(ev($racine_j - Rw_k$), *list*), *racine_set0*)**then** $l := l, k$
end if
end do;
$[l]$
end proc

$$\varepsilon := \frac{p1\, {a_2}^2\, {a_3}^2}{({a_2}^4 + 1)\, ({a_3}^4 + 1)}$$

$$Rcoef := \text{table}([(1,4) = -\frac{{a_3}^6\,{a_2}^6}{{p1}^2\,({a_2}^2\,{a_3}^2+1)\,(a_2\,a_3+1)\,(a_2\,a_3-1)},$$

$$(1,7) = -\frac{({a_2}^2+1)\,(a_2+1)\,(a_2-1)}{{a_2}^2},\ (1,5) = \frac{1}{p1},$$

$$(2,7) = \frac{({a_2}^2+1)^2\,(a_2+1)^2\,(a_2-1)^2}{{a_2}^4\,({a_2}^4+1)},$$

$$(2,4) = \frac{{a_2}^{16}\,{a_3}^{16}}{{p1}^4\,({a_2}^4\,{a_3}^4+1)\,({a_2}^2\,{a_3}^2+1)^2\,(a_2\,a_3+1)^2\,(a_2\,a_3-1)^2},\ (1,2) = -\frac{{a_3}^2}{p1},$$

$$(1,3) = \frac{{a_3}^2\,{a_2}^2}{p1},\ (1,1) = -\frac{({a_3}^2+1)\,(a_3+1)\,(a_3-1)}{{a_3}^2},$$

$$(2,1) = \frac{(a_3-1)^2\,(a_3+1)^2\,({a_3}^2+1)^2}{{a_3}^4\,({a_3}^4+1)},$$

$$(1,6) = -\frac{{a_2}^2}{p1}$$

$$])$$

Coefmo := **proc**(t)
local i, r, c;
  $c := 1$;
  $r := Rterme_t$;
  for i **to** 7 **do if** $0 < r_i$ **then** $c := c * Rcoef_{r_i,\,i}$ **end if end do**;
  $(-1)^{(r_2*r_3+r_2*r_5+r_2*r_6+r_3*r_5+r_3*r_6+r_5*r_6)} * c$
end proc

$f :=$ **proc**(l) $(-1)^{(l_2+l_6+l_3+l_5)}$ **end proc**

$Rmm := \mathbf{proc}(i,\ j)$

$\mathbf{local}\, ri,\ rj,\ s,\ eij,\ rc,\ t,\ mo,\ p,\ k;$

$\quad ri := racine_i\,;$

$\quad rj := racine_j\,;$

$\quad s := \mathrm{matrix}(17,\ 17,\ 0)\,;$

$\quad eij := \mathrm{matrix}(17,\ 17,\ 0)\,;$

$\quad eij_{i,\,j} := 1\,;$

$\quad rc := \mathrm{Rcontrib}(i,\ j)\,;$

$\quad \mathbf{for}\, t\, \mathbf{in}\, rc\, \mathbf{do}$

$\qquad mo := Rterm_t\,;$

$\qquad p := \mathrm{evalm}(\mathrm{Coefmo}(t) * eij)\,;$

$\qquad \mathbf{for}\, k\, \mathbf{to}\, \mathrm{nops}(mo)\, \mathbf{do}\, p := \mathrm{ev}(\text{‘}\&*\text{‘}(\text{‘}\&*\text{‘}(M_{mo_{-k}},\ p),\ tM_{7+mo_{-k}}))\, \mathbf{end\ do}\,;$

$\qquad \mathbf{if}\, parite_i = 0\ \ \mathbf{then}\, s := \mathrm{evalm}(s+p)\, \mathbf{else}\, s := \mathrm{evalm}(s + Rtpar_t * p)\, \mathbf{end\ if}$

$\quad \mathbf{end\ do};$

$\quad s := \mathrm{ev}({a_1}^{(-ri_1*rj_1)} * {a_2}^{(-ri_2*rj_2)} * {a_3}^{(-ri_3*rj_3)} * s)\,;$

$\quad \mathrm{ev}(s)$

$\mathbf{end\ proc}$

$$parite_mat := \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}$$

$\%1 := [1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1]$

$\%2 := [1,\ 1,\ 1,\ 1,\ -1,\ -1,\ 1,\ -1,\ -1,\ 1,\ 1,\ -1,\ -1,\ 1,\ -1,\ -1,\ 1]$

$Br\_0 := \textbf{proc}(ii,\, jj)$

$\textbf{local}\, m,\, i,\, j;$

$m := \mathrm{Rmm}(ii,\, jj)\,;$

$\mathrm{simp}([\mathrm{seq}(\mathrm{seq}([parite\_mat_{i,\,j} * m_{i,\,j},\, [j,\, i]],\, i = 1..17),\, j = 1..17)])$

end proc

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```
> save(M,tM,Brp,"UqD21-1.m");
> # Sauvegarde du résultat
> read "UqD21-1.m";
```

4 Opérateur de Tressage

```
> #(*****************************************************************
> )
> #(********************Opérateur de Tressage*************************)
> #(*****************************************************************)
> read "UqD21-1.m";
> BRp_0:=proc(t,i) local ta,tp;
> tp:=op(2,t); ta:=[op(1..(i-1),tp)];tp:=[op((i+2)..nops(tp),tp)];
> op(scal(op(1,t),
> map(proc(ti,ta,tp) [op(1,ti),[op(ta),op(op(2,ti)),op(tp)]]
> end,
> Brp[op(i,op(2,t)),op(i+1,op(2,t))],ta,tp))) end;
> # Le calcul de l'inverse Brn de Brp sera fait plus tard
> BRn_0:=proc(t,i) local ta,tp;
> tp:=op(2,t); ta:=[op(1..(i-1),tp)];tp:=[op((i+2)..nops(tp),tp)];
> op(scal(op(1,t),
> map(proc(ti,ta,tp) [op(1,ti),[op(ta),op(op(2,ti)),op(tp)]] end,
> Brn[op(i,op(2,t)),op(i+1,op(2,t))],ta,tp))) end;
> Braid:=proc(i,t) if i>0 then simp(map(BRp_0,t,i)) else
> simp(map(BRn_0,t,-i)) fi end;
> # Morphisme de tressage Id_L^(i-1)@c_{L,L}@Id_L^?      Si i<0
c'est
> # l'inverse du tressage
> parite_mat:=matrix(17,17,1):
> parite_mat:=ev(parite_mat-2*parite&*linalg[transpose](parite));
> Swap0:=proc(t,i) local k1,k2,t2;
> t2:=op(2,t); k1:=op(i,t2); k2:=op(i+1,t2);
> [parite_mat[k1,k2]*op(1,t),subsop(i=k2,i+1=k1,t2)] end;
> Swap:=proc(i,t)
> sort(map(Swap0,t,i),sndlex) end;
> # La volte "super"
> #(***********Vérification de la relation des tresses**************)
> # for i to 17 do
> #     for j to 17 do
> #         if (j=9) or (j=1) then print(i,j) fi;
> #         for k to 17 do l:=[i,j,k];
> #             Braid(1,Braid(2,Braid(1,[[1,l]])));
> #             Braid(2,Braid(1,Braid(2,[[1,l]])));
> #             if nops(map(factor,EqL(cl(1,-1,%,%%))))>0 then
> print(i,j,k) fi;
> #         od od od:
```

$BRp\_0 :=$ **proc**$(t,\ i)$
local $ta,\ tp;$
$tp := \mathrm{op}(2,\ t)\,;$
$ta := [\mathrm{op}(1..i-1,\ tp)]\,;$
$tp := [\mathrm{op}(i+2..\mathrm{nops}(tp),\ tp)]\,;$
$\mathrm{op}(\mathrm{scal}(\mathrm{op}(1,\ t), \mathrm{map}($
proc$(ti,\ ta,\ tp)\,[\mathrm{op}(1,\ ti),\ [\mathrm{op}(ta),\ \mathrm{op}(\mathrm{op}(2,\ ti)),\ \mathrm{op}(tp)]]$ **end proc**,
$Brp_{\mathrm{op}(i,\,\mathrm{op}(2,\,t)),\,\mathrm{op}(i+1,\,\mathrm{op}(2,\,t))},\ ta,\ tp)))$
end proc

$BRn\_0 :=$ **proc**$(t,\ i)$
local $ta,\ tp;$
$tp := \mathrm{op}(2,\ t)\,;$
$ta := [\mathrm{op}(1..i-1,\ tp)]\,;$
$tp := [\mathrm{op}(i+2..\mathrm{nops}(tp),\ tp)]\,;$
$\mathrm{op}(\mathrm{scal}(\mathrm{op}(1,\ t), \mathrm{map}($
proc$(ti,\ ta,\ tp)\,[\mathrm{op}(1,\ ti),\ [\mathrm{op}(ta),\ \mathrm{op}(\mathrm{op}(2,\ ti)),\ \mathrm{op}(tp)]]$ **end proc**,
$Brn_{\mathrm{op}(i,\,\mathrm{op}(2,\,t)),\,\mathrm{op}(i+1,\,\mathrm{op}(2,\,t))},\ ta,\ tp)))$
end proc

$Braid :=$ **proc**$(i,\ t)$
if $0 < i$ **then** $\mathrm{simp}(\mathrm{map}(BRp\_0,\ t,\ i))$ **else** $\mathrm{simp}(\mathrm{map}(BRn\_0,\ t,\ -i))$ **end if**
end proc

$$parite_mat := \begin{bmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&-1&-1&1&-1&-1&1&1&-1&-1&1&-1&-1&1\\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1
\end{bmatrix}$$

$\%1 := [1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1,\ 1]$
$\%2 := [1,\ 1,\ 1,\ 1,\ -1,\ -1,\ 1,\ -1,\ -1,\ 1,\ 1,\ -1,\ -1,\ 1,\ -1,\ -1,\ 1]$

$Swap0 := \mathbf{proc}(t,\, i)$

$\mathbf{local}\, k1,\, k2,\, t2;$

$\quad t2 := \mathrm{op}(2,\, t)\,;$

$\quad k1 := \mathrm{op}(i,\, t2)\,;$

$\quad k2 := \mathrm{op}(i+1,\, t2)\,;$

$\quad [parite\_mat_{k1,\, k2} * \mathrm{op}(1,\, t),\, \mathrm{subsop}(i = k2,\, i+1 = k1,\, t2)]$

$\mathbf{end\ proc}$

$Swap := \mathbf{proc}(i,\, t)\, \mathrm{sort}(\mathrm{map}(Swap0,\, t,\, i),\, sndlex)\ \ \mathbf{end\ proc}$

5 Recherche de la forme bilin invariante

```
> #(*****************************************************************
> )
> #(************Recherche de la forme bilin invariante*****************)
> #(*****************************************************************)
> # B:=matrix(17,17):  B[8,15]:=beta:
> # Bx:=proc() local l,i,j; l:=NULL;
> #     for i to 17 do for j to 17 do
> #                        if B[i,j]=evaln(B[i,j]) then l:=l,B[i,j]
fi
> #                  od
> #     od; {l} end;
> # for i to 3 do Eq[i]:=EqM(M[i]&*B&*M[i]-B) od:
> # solve(Eq[1] union Eq[2] union Eq[3] , Bx());
> # assign(%);
> # Eq[4]:=EqM(tM[4]&*B+M[3]^2&*B&*M[4]):
> # Eq[11]:=EqM(tM[11]&*B&*M[-3]^2+B&*M[11]):
> # Eq[10]:=EqM(tM[10]&*B+M[2]^2&*B&*M[10]):
> # Eq[17]:=EqM(tM[17]&*B&*M[-2]^2+B&*M[17]):
> # solve(Eq[4] union Eq[11] union Eq[10] union Eq[17] , Bx());
> # assign(%);
> # M[18]:=ev(M[1]&*M[-2]&*M[-3]):
> # M[-18]:=ev(M[-1]&*M[2]&*M[3]):
> # Eq[8]:=EqM(tM[8]&*B+M[18]&*MJ&*B&*M[8]):
> # Eq[15]:=EqM(tM[15]&*B&*M[-18]+MJ&*B&*M[15]):
> # solve(Eq[8] union Eq[15] , Bx());
> # assign(%);
> bilin:=matrix([
> [0, beta*(a[2]-1)*(a[2]+1)*(a[2]^2+1)/a[2]^2*gamma,
> beta*(a[3]-1)*(a[3]+1)*(a[3]^2+1)*gamma/a[3]^2,0,0,0,0,
> 0,0,0,0,0,0,0,0,0,0],
> [beta*(a[2]-1)*(a[2]+1)*(a[2]^2+1)/a[2]^2*gamma,
> beta*(a[2]-1)*(a[2]+1)*(a[2]^2+1)*(a[2]^4+1)*phi/a[2]^4,
> 0,0,0,0,0,0,0,0,0,0,0,0,0,0,0],
> [beta*(a[3]-1)*(a[3]+1)*(a[3]^2+1)*gamma/a[3]^2,0,
> beta*(a[3]^4+1)*(a[3]-1)*(a[3]+1)*(a[3]^2+1)*phi/a[3]^4,
> 0,0,0,0,0,0,0,0,0,0,0,0,0,0],
> [0,0,0,0,0,0,0,0,0,0,beta/a[3]^2,0,0,0,0,0,0],
> [0,0,0,0,0,0,0,0,0,0,0,beta/a[3]^2,0,0,0,0,0],
> [0,0,0,0,0,0,0,0,0,0,0,0,beta/a[2]^2/a[3]^2,0,0,0,0],
> [0,0,0,0,0,0,0,0,0,0,0,0,0,beta/a[2]^2/a[3]^2,0,0,0],
> [0,0,0,0,0,0,0,0,0,0,0,0,0,0,beta,0,0],
> [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,beta/a[2]^2,0],
> [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,beta/a[2]^2],
> [0,0,0,beta*a[3]^2,0,0,0,0,0,0,0,0,0,0,0,0,0],
> [0,0,0,0,-beta*a[3]^2,0,0,0,0,0,0,0,0,0,0,0,0],
> [0,0,0,0,0,-beta*a[2]^2*a[3]^2,0,0,0,0,0,0,0,0,0,0,0],
> [0,0,0,0,0,0,beta*a[2]^2*a[3]^2,0,0,0,0,0,0,0,0,0,0],
> [0,0,0,0,0,0,0,-beta,0,0,0,0,0,0,0,0,0],
> [0,0,0,0,0,0,0,0,-beta*a[2]^2,0,0,0,0,0,0,0,0],
> [0,0,0,0,0,0,0,0,0,beta*a[2]^2,0,0,0,0,0,0,0]]);
> Bil0:=proc(t,i) local ta,tp;
> tp:=op(2,t); if bilin[tp[i],tp[i+1]]=0 then NULL else
> [t[1]*bilin[tp[i],tp[i+1]],subsop(i+1=NULL,i=NULL,tp)] fi end;
> Bil:=proc(i,t)
> simp(map(Bil0,t,i)) end;
> # Morphisme d'evaluation  Id_L^(i-1) @ <.,.> @ Id_L^?
> Tens:=proc(l1,l2)
> simp(map(op, map(proc(m1,l2) (map(proc(m2,m1) [op(1,m1)*op(1,m2),
> [op(op(2,m1)),op(op(2,m2))]] end,l2,m1)) end,l1,l2))) end;
> # Produit tensoriel
```

$$
\mathit{bilin} :=
$$

$$
\left[0, \frac{\beta\,(a_2-1)\,(a_2+1)\,(a_2{}^2+1)\,\gamma}{a_2{}^2}, \frac{\beta\,(a_3-1)\,(a_3+1)\,(a_3{}^2+1)\,\gamma}{a_3{}^2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right]
$$

$$
\left[\frac{\beta\,(a_2-1)\,(a_2+1)\,(a_2{}^2+1)\,\gamma}{a_2{}^2}, \frac{\beta\,(a_2-1)\,(a_2+1)\,(a_2{}^2+1)\,(a_2{}^4+1)\,\phi}{a_2{}^4}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right]
$$

$$
\left[\frac{\beta\,(a_3-1)\,(a_3+1)\,(a_3{}^2+1)\,\gamma}{a_3{}^2}, 0, \frac{\beta\,(a_3{}^4+1)\,(a_3-1)\,(a_3+1)\,(a_3{}^2+1)\,\phi}{a_3{}^4}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right]
$$

$$
\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\beta}{a_3{}^2}, 0, 0, 0, 0, 0, 0\right]
$$

$$
\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\beta}{a_3{}^2}, 0, 0, 0, 0, 0\right]
$$

$$
\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\beta}{a_2{}^2\,a_3{}^2}, 0, 0, 0, 0\right]
$$

$$
\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\beta}{a_2{}^2\,a_3{}^2}, 0, 0, 0\right]
$$

$$
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \beta, 0, 0]
$$

$$
\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\beta}{a_2{}^2}, 0\right]
$$

$$
\left[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{\beta}{a_2{}^2}\right]
$$

$$
[0, 0, 0, \beta\,a_3{}^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$$

$$
[0, 0, 0, 0, -\beta\,a_3{}^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$$

$$
[0, 0, 0, 0, 0, -\beta\,a_2{}^2\,a_3{}^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$$

$$
[0, 0, 0, 0, 0, 0, \beta\,a_2{}^2\,a_3{}^2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$$

$$
[0, 0, 0, 0, 0, 0, 0, -\beta, 0, 0, 0, 0, 0, 0, 0, 0, 0]
$$

$$
[0, 0, 0, 0, 0, 0, 0, 0, -\beta\,a_2{}^2, 0, 0, 0, 0, 0, 0, 0, 0]
$$

$$
[0, 0, 0, 0, 0, 0, 0, 0, 0, \beta\,a_2{}^2, 0, 0, 0, 0, 0, 0, 0]
$$

$Bil0 := \mathbf{proc}(t,\, i)$
local *ta*, *tp*;
  $tp := \mathrm{op}(2,\, t)$;
  if $bilin_{tp_i,\, tp_{i+1}} = 0$ **then** *NULL*
  else $[t_1 * bilin_{tp_i,\, tp_{i+1}},\, \mathrm{subsop}(i+1 = NULL,\, i = NULL,\, tp)]$
  end if
end proc

$Bil := \mathbf{proc}(i,\, t)\, \mathrm{simp}(\mathrm{map}(Bil0,\, t,\, i))$ **end proc**

$Tens := \mathbf{proc}(l1,\, l2)$
  simp(map(*op*, map(**proc**(*m1*, *l2*)
    map(
    proc(*m2*, *m1*) [op(1, *m1*) ∗ op(1, *m2*), [op(op(2, *m1*)), op(op(2, *m2*))]] **end proc**, *l2*,
    m1)
  end proc, *l1*, *l2*)))
end proc

6 Recherche de la Coevaluation Omega \in L@L

```
> #(*****************************************************************
> )
> #(************Recherche de la Coevaluation Omega \in
> L@L*************)
> #(******************************************************************)
> # Omega:=[seq(seq([o[i,j],[i,j]],j=1..3),i=1..3),
> #         seq([o[i],[i,i+7]],i=4..10),seq([o[7+i],[i+7,i]],i=4..10)];
> # Eq:={}:
> # for i to 17 do
> #     Eq:=Eq union
> EqL(cl(1,-1,Bil(1,Tens([[1,[i]]],Omega)),[[1,[i]]])) od:
> # solve(Eq,{seq(seq(o[i,j],j=1..3),i=1..3),seq(o[i],i=4..17)});
> Omega:=[[1/2*Qn[3,4]*phi*Qn[2,4]/Qn[3,2]/Qn[2,2]/gamma^2/beta/Qn[1,2],
> [1, 1]],
> [-1/2*Qn[3,4]/Qn[3,2]/gamma/beta/Qn[1,2], [1, 2]],
> [-1/2*Qn[2,4]/Qn[2,2]/gamma/beta/Qn[1,2], [1, 3]],
> [-1/2*Qn[3,4]/Qn[3,2]/gamma/beta/Qn[1,2], [2, 1]],
> [-1/2*Qn[3,2]/beta/phi/Qn[1,2]/Qn[2,2], [2, 2]],
> [1/2*1/(Qn[1,2]*beta*phi), [2, 3]],
> [-1/2*Qn[2,4]/Qn[2,2]/gamma/beta/Qn[1,2], [3, 1]],
> [1/2*1/(Qn[1,2]*beta*phi), [3, 2]],
> [-1/2*Qn[2,2]/beta/phi/Qn[1,2]/Qn[3,2], [3, 3]],
> [1/(beta*a[3]^2), [4, 11]], [-1/(beta*a[3]^2), [5, 12]],
> [-1/(beta*a[2]^2*a[3]^2), [6, 13]], [1/(beta*a[2]^2*a[3]^2),
> [7, 14]],
> [-1/beta, [8, 15]], [-1/(beta*a[2]^2), [9, 16]],
> [1/(beta*a[2]^2), [10, 17]], [a[3]^2/beta, [11, 4]],
> [a[3]^2/beta, [12, 5]], [a[2]^2*a[3]^2/beta, [13, 6]],
> [a[2]^2*a[3]^2/beta, [14, 7]], [1/beta, [15, 8]],
> [a[2]^2/beta, [16, 9]], [a[2]^2/beta, [17, 10]]];
> Brn:=matrix(17,17,0):
> for i to 17 do print(i);
> for j to 17 do
> Brn[i,j]:=factor(Bil(3,Braid(2,Tens(Omega,[[1,[i,j]]]))))
> od od:
```

$$\Omega := \left[\left[\frac{1}{2} \frac{(a_3{}^4 - \frac{1}{a_3{}^4})\,\phi\,(a_2{}^4 - \frac{1}{a_2{}^4})}{\%1\,\%3\,\gamma^2\,\beta\,\%2}, [1, 1] \right], \left[-\frac{1}{2} \frac{a_3{}^4 - \frac{1}{a_3{}^4}}{\%1\,\gamma\,\beta\,\%2}, [1, 2] \right], \left[-\frac{1}{2} \frac{a_2{}^4 - \frac{1}{a_2{}^4}}{\%3\,\gamma\,\beta\,\%2}, [1, 3] \right], \right.$$

$$\left[-\frac{1}{2} \frac{a_3{}^4 - \frac{1}{a_3{}^4}}{\%1\,\gamma\,\beta\,\%2}, [2, 1] \right], [-\frac{1}{2} \frac{\%1}{\beta\,\phi\,\%2\,\%3}, [2, 2]], [\frac{1}{2} \frac{1}{\%2\,\beta\,\phi}, [2, 3]],$$

$$\left[-\frac{1}{2} \frac{a_2{}^4 - \frac{1}{a_2{}^4}}{\%3\,\gamma\,\beta\,\%2}, [3, 1] \right], [\frac{1}{2} \frac{1}{\%2\,\beta\,\phi}, [3, 2]], [-\frac{1}{2} \frac{\%3}{\beta\,\phi\,\%2\,\%1}, [3, 3]], [\frac{1}{\beta\,a_3{}^2}, [4, 11]],$$

$$[-\frac{1}{\beta\,a_3{}^2}, [5, 12]], [-\frac{1}{\beta\,a_2{}^2\,a_3{}^2}, [6, 13]], [\frac{1}{\beta\,a_2{}^2\,a_3{}^2}, [7, 14]], [-\frac{1}{\beta}, [8, 15]],$$

$$[-\frac{1}{\beta\,a_2{}^2}, [9, 16]], [\frac{1}{\beta\,a_2{}^2}, [10, 17]], [\frac{a_3{}^2}{\beta}, [11, 4]], [\frac{a_3{}^2}{\beta}, [12, 5]], [\frac{a_2{}^2\,a_3{}^2}{\beta}, [13, 6]],$$

$$\left. [\frac{a_2{}^2\,a_3{}^2}{\beta}, [14, 7]], [\frac{1}{\beta}, [15, 8]], [\frac{a_2{}^2}{\beta}, [16, 9]], [\frac{a_2{}^2}{\beta}, [17, 10]] \right]$$

$$\%1 := a_3{}^2 - \frac{1}{a_3{}^2}$$

$$\%2 := \frac{1}{a_2{}^2\,a_3{}^2} - a_2{}^2\,a_3{}^2$$

$$\%3 := a_2{}^2 - \frac{1}{a_2{}^2}$$

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17

7 Calcul du morphisme cocr : L->L@L

```
> #(*****************************************************************
> )
> #(*********Recherche d'un vecteur de plus haut poids [2,0,0]*********)
> #(*****************************************************************)
> # hv:=NULL: y:='y';
> # inco:=0; X:=proc() global inco; inco:=inco+1; y[inco] end;
> # for i to 17 do
> #     for j to 17 do
> #         if (convert(ev(racine[i]+racine[j]),list)=[2,0,0])
> #         then hv:=hv,[X(),[i,j]] fi od od:
> # hv:=[hv];
> # Mhv:=matrix(17,17,0):
> # for m in hv do Mhv[op(m[2])]:=m[1] od:
> # Eq:='Eq'; y[9]:=-2*delta;
> # Eq[4]:=EqM(M[4]&*Mhv+M[3]^2&*Mhv&*tM[4]):
> # Eq[10]:=EqM(M[10]&*Mhv+M[2]^2&*Mhv&*tM[10]):
> # Eq[8]:=EqM(M[8]&*Mhv+M[18]&*MJ&*Mhv&*tM[8]):
> # solve(Eq[4] union Eq[10] union Eq[8], {seq(y[i],i=1..8),y[10]});
> # assign(%);
> hv:=[[phi*delta*(a[2]^4*a[3]^4+a[3]^4+a[2]^4+1)/gamma/a[2]^2/a[3]^2,
> [1, 7]],
> [-(a[3]^4+1)*delta/a[3]^2, [2, 7]], [-(a[2]^4+1)*delta/a[2]^2,
> [3, 7]],
> [2*delta/a[3]^2, [5, 9]], [-2/a[2]^2/a[3]^2*delta, [6, 8]],
> [-phi*delta*(a[2]^4*a[3]^4+a[3]^4+a[2]^4+1)/gamma/a[2]^2/a[3]^2,
> [7, 1]],
> [(a[3]^4+1)*delta/a[3]^2, [7, 2]], [(a[2]^4+1)*delta/a[2]^2,
[7,
> 3]],
> [-2*delta, [8, 6]], [2*delta/a[2]^2, [9, 5]]]:
```

```
> #(*****************************************************************
> )
> #(*********Calcul du morphisme cocr :  L->L@L ***********************)
> #(*****************************************************************)
> CoM[7]:=matrix(17,17,0):  for m in hv do CoM[7][op(m[2])]:=m[1]
od:
> CoM[6]:=ev((M[15]&*CoM[7]&*M[-18]+MJ&*CoM[7]&*tM[15])/tM[15][7,6]):
> CoM[9]:=ev((M[11]&*CoM[6]&*M[-3]^2+CoM[6]&*tM[11])/tM[11][6,9]):
> CoM[5]:=ev((M[17]&*CoM[6]&*M[-2]^2+CoM[6]&*tM[17])/tM[17][6,5]):
> CoM[8]:=ev((M[17]&*CoM[9]&*M[-2]^2+CoM[9]&*tM[17])/tM[17][9,8]):
> CoM[10]:=ev((M[15]&*CoM[9]&*M[-18]+MJ&*CoM[9]&*tM[15])/tM[15][9,10]):
> CoM[4]:=ev((M[15]&*CoM[5]&*M[-18]+MJ&*CoM[5]&*tM[15])/tM[15][5,4]):
> CoM[1]:=ev((M[15]&*CoM[8]&*M[-18]+MJ&*CoM[8]&*tM[15])/tM[15][8,1]):
> CoM[11]:=ev((M[11]&*CoM[1]&*M[-3]^2+CoM[1]&*tM[11])/tM[11][1,11]):
> CoM[17]:=ev((M[17]&*CoM[1]&*M[-2]^2+CoM[1]&*tM[17])/tM[17][1,17]):
> CoM[3]:=ev((M[11]&*CoM[4]&*M[-3]^2+CoM[4]&*tM[11])/tM[11][4,3]):
> CoM[2]:=ev((M[17]&*CoM[10]&*M[-2]^2+CoM[10]&*tM[17])/tM[17][10,2]):
> CoM[15]:=ev((M[15]&*CoM[2]&*M[-18]+MJ&*CoM[2]&*tM[15])/tM[15][2,15]):
> CoM[16]:=ev((M[17]&*CoM[15]&*M[-2]^2+CoM[15]&*tM[17])/tM[17][15,16]):
> CoM[12]:=ev((M[11]&*CoM[15]&*M[-3]^2+CoM[15]&*tM[11])/tM[11][15,12]):
> CoM[13]:=ev((M[17]&*CoM[12]&*M[-2]^2+CoM[12]&*tM[17])/tM[17][12,13]):
> CoM[14]:=ev((M[15]&*CoM[13]&*M[-18]+MJ&*CoM[13]&*tM[15])/tM[15][13,14]
> ):
> cocr:=vector(17,0):
> for k to 17 do
> cocr[k]:=simp([seq(seq([CoM[k][i,j],[i,j]],j=1..17),i=1..17)])
od:
> Cocr0:=proc(t,i) local ta,tp;
> tp:=op(2,t);
> ta:=[op(1..(i-1),tp)];
> tp:=[op((i+1)..nops(tp),tp)];
> op(scal(op(1,t),
> map(proc(ti,ta,tp) [op(1,ti),[op(ta),op(op(2,ti)),op(tp)]] end,
> cocr[op(i,op(2,t))],ta,tp))) end;
> Cocr:=proc(i,t)
> simp(map(Cocr0,t,i)) end;
> # Morphisme Id_L^(i-1) @ cocr @ Id_L^?
```

Cocr0 := **proc**(t, i)
local ta, tp;
  $tp := \mathrm{op}(2, t)$;
  $ta := [\mathrm{op}(1..i-1, tp)]$;
  $tp := [\mathrm{op}(i+1..\mathrm{nops}(tp), tp)]$;
  op(scal(op(1, t), map(
    proc(ti, ta, tp) [op(1, ti), [op(ta), op(op(2, ti)), op(tp)]] **end proc**,
    $cocr_{\mathrm{op}(i,\,\mathrm{op}(2,\,t))}$, ta, tp)))
end proc

Cocr := **proc**(i, t) simp(map(*Cocr0*, t, i)) **end proc**

```
> #(****************************************************************
> )
> #(*********Calcul du morphisme crochet :  L@L->L ********************)
> #(****************************************************************)
> crochet:=matrix(17,17,0):
> for i to 17 do
> for j to 17 do
> crochet[i,j]:=Bil(1,Tens([[1,[i]]],cocr[j])) od od:
> Cr0:=proc(t,i) local ta,tp;
> tp:=op(2,t);
> ta:=[op(1..(i-1),tp)];
> tp:=[op((i+2)..nops(tp),tp)];
> op(scal(op(1,t),
> map(proc(ti,ta,tp) [op(1,ti),[op(ta),op(op(2,ti)),op(tp)]] end,
> crochet[op(i,op(2,t)),op(i+1,op(2,t))],ta,tp))) end;
> Cr:=proc(i,t)
> simp(map(Cr0,t,i)) end;
> # Morphisme Id_L^(i-1) @ crochet @ Id_L^?
```

Cr0 := **proc**(t, i)
local ta, tp;
$tp := \mathrm{op}(2, t)$;
$ta := [\mathrm{op}(1..i-1, tp)]$;
$tp := [\mathrm{op}(i+2..\mathrm{nops}(tp), tp)]$;
op(scal(op(1, t), map(
proc(ti, ta, tp) [op(1, ti), [op(ta), op(op(2, ti)), op(tp)]] **end proc**,
$crochet_{\mathrm{op}(i,\,\mathrm{op}(2,\,t)),\,\mathrm{op}(i+1,\,\mathrm{op}(2,\,t))}$, ta, tp)))
end proc

Cr := **proc**(i, t) simp(map(*Cr0*, t, i)) **end proc**

```
> save(racine, parite, parite_mat, listorder, sndlex, scal, simp,
vide,
> cl,
> EqL, EqM, ev, ttrace, qn, M, tM, Brp, Brn, BRp_0, BRn_0, Braid,
> Swap0,
> Swap, bilin, Bil0, Bil, Tens, Omega, cocr, Cocr0, Cocr, crochet,
> Cr, Cr0,
> "UqD21-2.m");
> read("UqD21-2.m");
```

8 Relations skein

```
> # Pour fixer les paramètres :
> phi:=-Phi/2/beta/delta^2;
> a[1]:=1/a[2]/a[3];
> mono:=proc(i) [[1,[i]]] end;
> with(linalg):
> quant:=proc(x) local y;
> y:=factor(x);
> y:=factor(subs((a[2]^4*a[3]^4+1)=(a[2]^4*a[3]^4+1)*Q[1,4]/qn(1,4),y));
> y:=factor(subs((a[2]^4+1)=(a[2]^4+1)*Q[2,4]/qn(2,4),y));
> y:=factor(subs((a[3]^4+1)=(a[3]^4+1)*Q[3,4]/qn(3,4),y));
> y:=factor(subs((a[2]^2*a[3]^2+1)=(a[2]^2*a[3]^2+1)*Q[1,2]/qn(1,2),y));
> y:=factor(subs((a[2]^2+1)=(a[2]^2+1)*Q[2,2]/qn(2,2),y));
> y:=factor(subs((a[3]^2+1)=(a[3]^2+1)*Q[3,2]/qn(3,2),y));
> y:=factor(subs((a[2]*a[3]+1)=(a[2]*a[3]+1)*Q[1,1]/qn(1,1),y));
> y:=factor(subs((a[2]+1)=(a[2]+1)*Q[2,1]/qn(2,1),y));
> y:=factor(subs((a[3]+1)=(a[3]+1)*Q[3,1]/qn(3,1),y));
> y end;
> unprotect(Psi);
> Psi:=proc(t) Bil(2,Cocr(1,Cocr(2,t))) end;
> mono:=proc(i) [[1,[i]]] end;
> rap:=proc(l,ll) local c; global Er;
> c:=factor(op([1,1],l)/op([1,1],ll));
> Er:=cl(1,-c,l,ll); print(Er); c end;
> # pour comparer deux tenseurs
```

$$\phi := -\frac{\Phi}{2\,\beta\,\delta^2}$$

$$a_1 := \frac{1}{a_2\,a_3}$$

$$\mathit{mono} := \mathbf{proc}(i)\,[[1,\,[i]]]\ \ \mathbf{end\ proc}$$

$\mathit{quant} := \mathbf{proc}(x)$
$\mathbf{local}\, y;$
$\quad y := \mathrm{factor}(x)\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}({a_2}^4 * {a_3}^4 + 1 = ({a_2}^4 * {a_3}^4 + 1) * Q_{1,\,4}/\mathrm{qn}(1,\,4),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}({a_2}^4 + 1 = ({a_2}^4 + 1) * Q_{2,\,4}/\mathrm{qn}(2,\,4),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}({a_3}^4 + 1 = ({a_3}^4 + 1) * Q_{3,\,4}/\mathrm{qn}(3,\,4),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}({a_2}^2 * {a_3}^2 + 1 = ({a_2}^2 * {a_3}^2 + 1) * Q_{1,\,2}/\mathrm{qn}(1,\,2),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}({a_2}^2 + 1 = ({a_2}^2 + 1) * Q_{2,\,2}/\mathrm{qn}(2,\,2),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}({a_3}^2 + 1 = ({a_3}^2 + 1) * Q_{3,\,2}/\mathrm{qn}(3,\,2),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}(a_2 * a_3 + 1 = (a_2 * a_3 + 1) * Q_{1,\,1}/\mathrm{qn}(1,\,1),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}(a_2 + 1 = (a_2 + 1) * Q_{2,\,1}/\mathrm{qn}(2,\,1),\,y))\,;$
$\quad y := \mathrm{factor}(\mathrm{subs}(a_3 + 1 = (a_3 + 1) * Q_{3,\,1}/\mathrm{qn}(3,\,1),\,y))\,;$
$\quad y$
$\mathbf{end\ proc}$

$$\Psi := \mathbf{proc}(t)\,\mathrm{Bil}(2,\,\mathrm{Cocr}(1,\,\mathrm{Cocr}(2,\,t)))\,\mathbf{end\ proc}$$

$mono := \mathbf{proc}(i)\ [[1,\ [i]]]\ \ \mathbf{end\ proc}$

$rap := \mathbf{proc}(l,\ ll)$
local c;
global Er;
$c := \mathrm{factor}(\mathrm{op}([1,\ 1],\ l)/\mathrm{op}([1,\ 1],\ ll))\,;\ Er := \mathrm{cl}(1,\ -c,\ l,\ ll)\,;\ \mathrm{print}(Er)\,;\ c$
end proc

```
> #(*******************************************************************
> )
> #(**********************Relations skein****************************)
> #(*****************************************************************)
> rap(Braid(1,Omega),Omega); #-> 1
> W:=Cocr(1,Omega):  rap(Braid(1,W),W); #-> -1
> Cr(1,W); #-> []
> O2:=Tens(Omega,Omega):
> TangBp:=Braid(2,O2):  TangBm:=Braid(-2,O2):
> TangI:=Cocr(2,Cr(2,O2)):  TangH:=Cr(2,Cocr(3,O2)):
> TangXp:=Cr(2,Cocr(3,TangBp)):  TangXm:=Cr(2,Cocr(3,TangBm)):
> rap(cl(1,-1,TangBp,TangBm),
> cl(1,1/2,cl(1,-1,TangH,TangI),
> cl(1,1,TangXp,TangXm))); quant(%);
> # quantum IHX relation
> #-> -2*a[2]^4*a[3]^4/Phi/(a[3]^4+1)/(a[2]^4+1)/(a[2]^4*a[3]^4+1)=
> #        -2*Q[1,2]*Q[2,2]*Q[3,2]/Q[1,4]/Q[2,4]/Q[3,4]/Phi
> rap(Cr(2,TangH),W); quant(%); #-> -Q[1,2]*Q[2,2]*Q[3,2]*Phi
> rap(Cr(1,Braid(2,TangBp)),W); quant(%); #-> 2*Q[1,2]*Q[2,2]*Q[3,2]
```

$$[]$$
$$1$$
$$[]$$
$$-1$$
$$[]$$
$$[]$$
$$-\frac{2\,{a_2}^4\,{a_3}^4}{\Phi\,({a_3}^4+1)\,({a_2}^4+1)\,({a_2}^4\,{a_3}^4+1)}$$
$$-\frac{2\,Q_{2,\,2}\,Q_{1,\,2}\,Q_{3,\,2}}{Q_{1,\,4}\,Q_{2,\,4}\,Q_{3,\,4}\,\Phi}$$
$$[]$$
$$\frac{\Phi\,(a_2-1)\,(a_2+1)\,({a_2}^2+1)\,(a_3-1)\,(a_3+1)\,({a_3}^2+1)\,(a_2\,a_3-1)\,(a_2\,a_3+1)\,({a_2}^2\,{a_3}^2+1)}{{a_2}^4\,{a_3}^4}$$
$$-Q_{1,\,2}\,Q_{3,\,2}\,Q_{2,\,2}\,\Phi$$
$$[]$$
$$-\frac{2\,(a_2\,a_3-1)\,(a_2\,a_3+1)\,({a_2}^2\,{a_3}^2+1)\,({a_2}^2+1)\,(a_2+1)\,(a_2-1)\,(a_3-1)\,(a_3+1)\,({a_3}^2+1)}{{a_2}^4\,{a_3}^4}$$

$$2\,Q_{3,\,2}\,Q_{2,\,2}\,Q_{1,\,2}$$