Introducing some sustainable development notions to students of Mathematics using a nested cycles model

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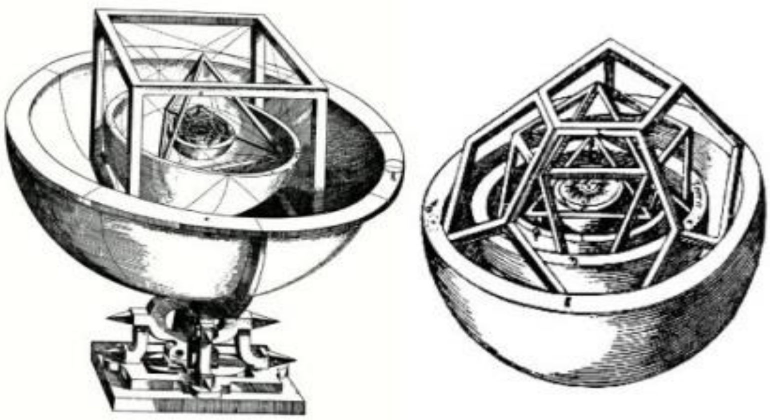


Johannes Kepler – Mysterium Cosmographicum (1660)

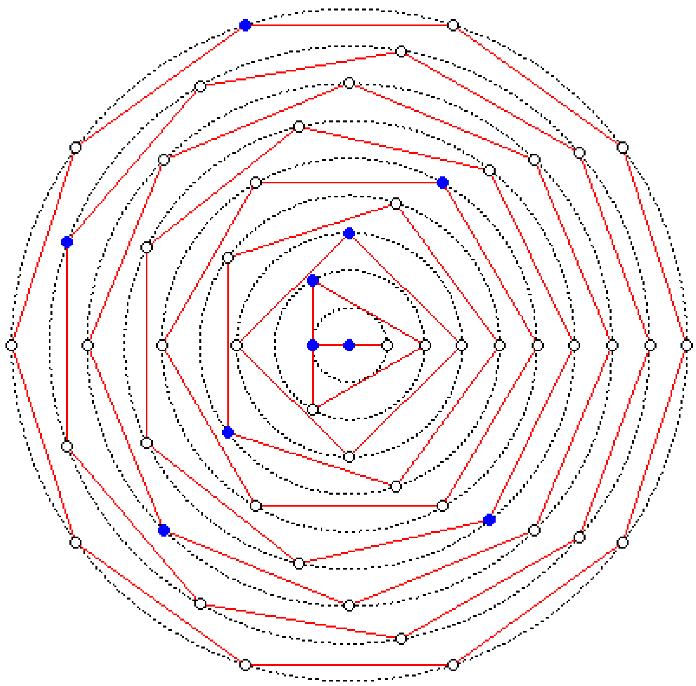
Natural Cycles

We model the aspects of Nature which are relevant to a phenomenon of interest as an infinity of nested regular polygons with 1, 2,…, n,… vertices, inscribed in circles of radii 0, 1, 2,…, n-1,… . Each polygon represents a *natural cycle* and each vertex a *state* of the cycle.

If the *time scale* is one year, the polygon with one vertex (a point) corresponds to an annual cycle. The polygon with two vertices corresponds to a biannual cycle,…, the polygon with 10 vertices to a decennial cycle, etc.

One can use this model to visualize the seasonal decomposition of a time series with no trend; in such a case, one can associate to each cycle a *seasonal coefficient*.

More generally, we can assume that a weight vector is associated to each cycle. A cycle will be said to be *active* if its weight vector is not null. This means that the cycle has an impact on the phenomenon of interest.



Ten nested natural cycles in a natural configuration

The Environment

The effect of environmental processes on the phenomenon of interest is modelled by a rotation of each n-polygon by an angle of $2\pi/n$. Such a rotation will be called a *standard rotation*. It corresponds to the effects of environmental process during one period of time (for instance, a year).

The group generated by the standard rotation (a cyclical group of order n) on the regular polygon with n vertices will be called the *environment* of the natural cycle of order n.

The environment of a cycle will be said to be *active* if the cycle is active.

The *Global Environment* is defined as the direct product (in the mathematical sense) of all the active environments.

The global environment *adapts* to each natural cycle; it *acts* on each cycle (in the mathematical sense) through a subgroup which is isomorphic to the corresponding cyclical group.

Natural States

We assume that only the first N cycles are active. We start from an initial configuration of vertices – we shall say that the vertices are aligned – and we make it evolve under the action of the standard rotations associated to the different cycles. Thus, we obtain a sequence of configurations that will be called *natural configurations* or *natural states*. Since we limit ourselves to the first N cycles, there are N ! configurations which are possible theoretically, but only l.c.m.(1,2,…,N) natural states (where l.c.m. denotes the least common multiple). When N gets bigger and bigger, the ratio between the number of natural states and the number of theoretically possible configurations tends to 0, meaning that the natural states are increasingly rare. Indeed, one can prove that l.c.m(1,2,..,N) < 3^N [1].

To each natural state, one can associate for instance the abundance of a natural *resource*, be it animal or vegetal, at the beginning of the corresponding year.

Environmental Complexity

We can quantify the *complexity* of the Global Environment as the sum of the number of vertices of the active cycles, that is, as the sum of the orders of all the cyclical groups appearing in the Global Environment.

For instance, if only the N first cycles are active, the environmental complexity is equal to N (N+1)/2.

If only the first and the third natural cycles are active, it is equal to 4.

An open mathematical problem consists in determining the maximal number of natural states for a given environmental complexity [2].

Perturbations of the Environment

We shall call a *perturbation* of the Global Environment any action which leads to the appearance or disappearance of some natural cycles.

Such a perturbation can be obtained by applying a non-standard rotation, with angle a *rational* multiple of 2π, to a cycle, starting from a given year. This leads to the disappearance and appearance of one cycle at most.

Now assume that the action of the Environment on the cycles depends on a variable z. It can happen that the standard rotation is applied if $z < s$ and is not applied if $z > s$. In such a case, *s* is called a *threshold*.

Observables

We shall call an *observable* any random variable x whose distribution depends on the global configuration of the active cycles (for instance, the quantity of fish harvested in a year in a given area). If the natural configuration in year i is denoted by i, then the *stock* of the observable x for year i is given by x(i) and its *flux* is given by x(i)-x(i-1) (for $i > 1$).

An application

In the framework of our model, how can one exploit a natural resource in a sustainable way ?

We assume that a finite number of cycles is active for the resource of interest.

We assume the resource's stock at the end of a period of time (a year, for instance) depends only on the resource's stock and the cycles configuration at the beginning of the period.

We assume that the environment of one of the cycles is *controlled*, that is, that we can choose the rotation which is applied on the cycle during the period of time among the rotations generated by the standard rotation.

We assume that this control has no influence on the other cycles.

We shall say that an exploitation strategy is *strongly sustainable* if Nature carries on when the exploitation of the resource stops, that is, if the modifications made are *reversible*. We shall say that the strategy is weakly sustainable if it is *transparent*, that is, if its influence on the resource cannot be detected from the evolution of the resource's stock.

We shall show that is possible, in the framework of our model, to extract a profit while exploiting the resource sustainably, in the weak and strong senses.

At the beginning of a period of time, we identify the configuration of all the cycles. We shall call the configuration of *all the cycles except the controlled cycle* the *context*. We then look among the *natural configurations* the configurations with the same context. If such configurations exist, we choose the one which provides, at the end of one period of time, the highest possible stock that is higher than the one which would have been obtained naturally. We then act on the controlled cycle in order to reach this configuration. One period of time later, we harvest the difference between the available stock and the one that would have been obtained naturally. Finally, we act on the controlled cycle so as to bring it to the state that it would have occupied naturally at the end of the period.

Following this strategy, the resource's stock at the beginning of each period of time is equal to what would have obtained naturally, that is without control and harvesting. This means that the proposed strategy is *weakly* sustainable. Moreover of one stops exploiting the resource, it resumes its natural evolution since we have jointly, at the beginning of each period of time, a natural configuration and a stock which is equal to what would have been obtained naturally. Thus, our strategy is also *strongly* sustainable.

A *profit* is extracted if the profits made from commercializing the harvest are higher than the costs of acting on the controlled cycle.

References

[1] D. Hanson. *On the Product of the Primes*, Canad. Math. Bull. Vol. 15 (1), 1972.

[2] J.-L. Nicolas. *Ordre Maximal d'un élément du Groupe Sn des permutations et « highly composite numbers »*. Bulletin de la Société Mathématique de France, Tome 97 (1969).

[3] J. S. Rose*.* *A Course on Group Theory*. Dover, 2012.