A Compact and Multiscale Image Model Based on Level Sets

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Abstract. Multiscale segmentation respectful of the visual perception is an important issue of Computer Vision. We present an image model derived from the level sets representation which offers most of the properties sought to a good segmentation : the borders are located at the perceptual edges; they are invariant by affine map and by contrast change; they are sorted according to their perceptual significance using a scale parameter. At last, a compact version of this model has been developed to be used in a progressive, and artifact-free, image compression scheme.

1 Introduction

One of the basic problem of image analysis is to define a mathematical representation that offers suitable properties upon which subsequent computer vision algorithms would operate.

The edge detection theory of Hildreth and Marr [19] [26] was one of the first attempts to solve this problem using a multiscale analysis. The *raw primal sketch* of D.Marr is based on the detection of the intensity changes in the image, by recording the zero-crossing location of the image filtered by the Laplacian of the Gaussian at a given scale. The edges are then defined as discontinuity lines, and the scale parameter allows to discriminate the important *atoms*. This approach has been successfully developed in the past, since it meets almost all the requirements of a "good primal sketch" (see e.g. [10] for an optimal edge detector). These last years have seen interesting reformalizations of this multiscale edges representation, in a wavelet [18] and in a variational [21] framework.

However, this approach still suffers of some drawbacks that do not make it always suitable for some processes : the representation is not invariant under contrast changes. This means that the edge locations of an image on which a contrast change has been applied differ from the original edge locations. It is well known since M.Wertheimer [25] that the visual perception of edges does not depend of the light level. Therefore, edges should not be computed using a discrete derivative. This deviation is a major inconvenience for pattern recognition processes, but it does not really apply to compression problem. This last domain is concerned by the completeness of the representation. Although the classical linear multiscale edges representation is mathematically not complete, algorithms which allow to reconstruct an image close to the original have been described [14] [18]. But when the representation is altered by a compression process, visual artifacts (e.g. Gibbs phenomena) appear on the reconstructed image.

More recently, it has been proved in [2] that, under fairly conditions (including invariance under change of contrast, under change of scale and under affine map), there exists only one regular multiscale analysis, the so-called AMSS (for Affine Morphological Scale Space). An image is decomposed into this scale-space using a parabolic evolution equation, for which viscosity solutions [9] exist. Because of the morphological invariance, the evolution of the image along the scales is equivalent to the evolution of its level curves, which are defined as the border of the level sets. A level set is a set of pixels with gray levels below (or above) a given threshold.

The representation of an image by its level sets has been proposed by the Mathematical Morphology school [22] as a geometrical decomposition which offers the contrast invariance. This representation can be viewed as another raw primal sketch, for which all good properties are met but the compactness. Recently, such a decomposition based on the connected components of the level lines has been described [20], together with a fast algorithm. This representation is well adapted to number of image analysis problems (as pattern matching) but it still suffers of a relatively large amount of data. Using AMSS, a simplification of the image can be performed to reduce the amount of data. However, the structure of the image is then considerably weakened and the filtered image does not sound natural.

In this paper, we introduce another model based on the level sets that can be coded using a very small amount of data. We propose in Section 2 a new definition of morphological edges, based on the selection of the most perceptive level sets. We use the conjecture expressed in [6] about the atoms of the visual perception, made by pieces of level lines joining junctions. The regions formed by these level sets compose a segmentation at a given scale. Section 3 describes our image model : using an elliptic PDE [7], the image is approximated by smooth and non-oscillating functions on each regions given by the segmentation. This defines a sketch image which carries the most important structures, up to the scale of the less perceptive edges. According to the quantity of pixel's values retained in the border's regions, the image model may be used as a compact image representation. A straightforward application of this compact model is the design of a compression scheme that respects the human visual system. We illustrate this capacity in Section 4.

2 Morphological edges and multiscale segmentation

This section addresses the problem of computing a multiscale and morphological segmentation $\mathcal{P} = \{P_i / i = 1, \ldots, n\}$ such that the borders of the most important structures match the borders of the regions P_i. A classical answer would be to try to extract the edges, in the sense of the discontinuity lines in the image. However, the use of classical edge detectors is not consistent with the morphological approach, as it is well explained in [6], essentially because such edges are not contrast invariant. In this paper, V.Caselles, B.Coll and J.M.Morel argue that the atoms of the perception, that is, the basic elements on which further representations may be built, are not edges but "pieces of level lines joining junctions".

Let us first recall what level lines are. Let Ω be an open bounded subset of $\mathbb{R}^2$. The total variation of an image $u : \Omega \to \mathbb{R}$ can be simply defined, if $u \in C^1(\Omega)$, as

$$\mathrm{TV}(u) = \int_\Omega |\nabla u(x)| \, dx. \tag{1}$$

If the gradient of u, ∇u, does not exist or is not continuous but if $u \in L^1(\Omega)$, (1) is generalized into

$$\mathrm{TV}(u) = \sup_\phi \{ \int_\Omega u(x)(\mathrm{div}\phi)(x) \, dx \, / \, \phi \in C_c^1(\Omega, \mathbb{R}^2) \text{ and } |\phi| \leq 1 \}. \tag{2}$$

We say that u is of bounded variation ($u \in \mathrm{BV}(\Omega)$) if $\mathrm{TV}(u) < +\infty$. A set $P \subset \Omega$ has finite perimeter if $\mathrm{per}(P) = \mathrm{TV}(\mathbb{1}_P) < +\infty$. In that case $\partial^* P$, the essential boundary of P, is an union of a countable set of Jordan curves [8].

Let L_λ be the lower level set λ and M_μ the upper level set μ of u:

$$L_\lambda = \{x \in \Omega / u(x) \leq \lambda\}, M_\mu = \{x \in \Omega / u(x) \geq \mu\}. \tag{3}$$

We shall call level set any lower or upper level set. The family $(L_\lambda)_\lambda$ or $(M_\mu)_\mu$ is a complete representation, since one can reconstruct the image by

$$u(x) = \inf_\lambda \{x \in L_\lambda\} = \sup_\mu \{x \in M_\mu\}. \tag{4}$$

If u is BV, then all level sets are of finite perimeter and their essential boundaries constitute the level lines of u. If we map the level lines of an image for a given set of levels $\{\lambda_1 < \lambda_2 \ldots < \lambda_n = +\infty\}$, we get a segmentation of the image with sets of type $\{x \in \Omega / \lambda_{i-1} < u(x) < \lambda_i\}$, also called topographic map [6] (see first line of Figure 3). More generally, one can consider a segmentation achieved using only some connected components of lower levet sets $(L_\lambda)_\lambda$ and upper level sets $(M_\mu)_\mu$. Notice that pieces of some (but not all) level lines are located at the perceptive edges and that conversely, all perceptive edges correspond to pieces of some level lines. A topographic map has also interesting invariance properties : the map commutes with any affine transformation performed on the image (translation, rotation, and zoom) and it does not change when the contrast of the image is modified (the so-called morphological property). Thus, a topographic map achieves a morphological segmentation with suitable properties to build an image model based on perceptual edges. The question is now : how should we select the level sets so that the level lines match as well as possible the visual perception of edges ?

We shall emphasize that the physical generation process of an image implies some events (as occlusions and transparencies) which cause singularities on the topographic map : level lines joining some other level lines with a shape (more or less) like a T in case of an occlusion. The T-junction singularity is one of the most significant principles of the visual reconstruction, which allows a geometrical constitution of the visual objects. It is in the heart of the Gestaltist's theory, and in particular of the Kanizsa's work [15] [16]. Each time a T-junction is detected, our perception reconstructs the occlusion of an object by another one, and the border of the occluded object is mentally extended behind the horizontal bar of the T. In the left drawing of Figure 1, the observer reconstructs black disks from quarters of disks only. This phenomenological description, originally formulated by G.Kanizsa in the case of drawings, can be easily adapted to digital images using level lines [1]. The main difference lies in the fact that on drawings, T-junctions occur where the line of the pen meets a previous line only, that is, at places where an object begins to come in front of another. On digital images of natural world, the objects are never uniformly shined, and therefore even unvaried colored surfaces present lot of level lines. At the borders of an object, these level lines meet the level lines of the background and generate multiple T-junctions : occlusions occur along all the borders. The shapes of the objects is then essentially characterised by the T-junctions on them, and by the pieces of the level lines joining these junctions. In this way, one may give a morphological definition of edges : we call *morphological edge* a piece of level line joining any number of T-junctions. The more a morphological edge contains T-junctions, the more it is perceptually significant : the number of T-junctions contained in a morphological edge behaves as an inverse scale parameter.

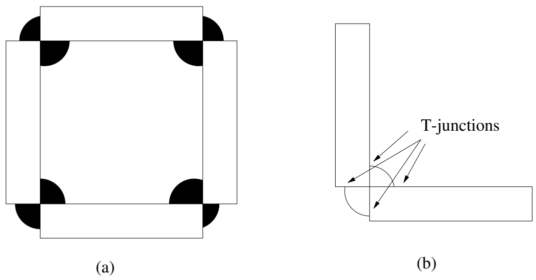

Fig. 1. The visual power of T-junctions. The human visual system reconstructs the black objects of the drawing (a) as disks partially covered by boxes. This reconstruction is due to the T-junctions, which are abvious on the topographic map (b). One of the Kanizsa's principle says that the border of the occluded object has to be extended so that to preserve its curvature.

To detect the significant T-junctions on natural images, we use an algorithm adapted from [6] which ensures the existence of three connected components with non-negligible size, one belonging to the occulting object, one belonging to the occulted object and one part of the background. The geometry of every recorded T-junction is characterized by only two of these three connected components. For our image model, we consider the one wich is a lower level set L_λ and the one which is an upper level set M_μ (see Figure 2). In this way, morphological edges are composed by borders of connected components denoted L^k_λ and M^k_μ. Notice that in the discrete case $\Omega \subset \mathbb{Z}^2$, the border of a region lies in the shifted grid $(\mathbb{Z}+1/2)^2$. If we note ∂P the internal border of a region P (which lies in the $\mathbb{Z}^2$ grid), at every T-junction $x \in (\mathbb{Z}+1/2)^2$ can be associated the neighbor pixels $x^k_\lambda \in \partial L^k_\lambda$ and $y^k_\mu \in \partial M^k_\mu$.

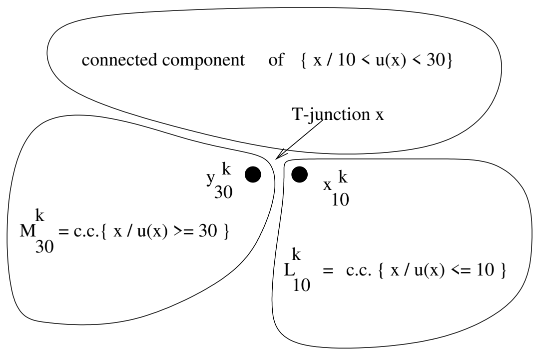

Fig. 2. The T-junction detection algorithm ensures the existence of three significant connected components, one being part of a lower level set L_λ and one being part of an upper level set M_μ (in this example, $\lambda = 10$ and $\mu = 30$). The T-junction point x does not belong to the pixel's grid, but the points x^k_λ and y^k_μ do.

Since each morphological edge belongs to a border of a set L^k_λ or M^k_μ, the issue is to choice, from all possible connected components of the level sets family $(L_\lambda)_\lambda$ and $(M_\mu)_\mu$, the ones that contain the greatest numbers of T-junctions :

Multiscale segmentation algorithm -

Parameter : $s \in [0,1]$ is the scale of the segmentation.

Step 1 : decompose the image u into its level sets $(L_\lambda)_\lambda$ and $(M_\mu)_\mu$.

Step 2 : compute the sequences of T-junctions $T_L = (x^k_\lambda)_{\lambda,k}$, $T_M = (y^k_\mu)_{\mu,k}$ and the associated connected components L^k_λ and M^k_μ. Let N be the number of T-junctions : $N = |T_L| = |T_M|$.

Step 3 : sort the sequences L^k_λ and M^k_μ in the order given by the number of T-junctions : L^k_λ is before $L^{k'}_{\lambda'}$ if $|T_L \cap \partial L^k_\lambda| > |T_L \cap \partial L^{k'}_{\lambda'}|$ and M^k_μ is before $M^{k'}_{\mu'}$ if $|T_M \cap \partial M^k_\mu| > |T_M \cap \partial M^{k'}_{\mu'}|$.

Step 4 : the multiscale segmentation $\mathcal{P}$ is made by the first $N(1-s)$ connected components of the sequences L^k_λ and M^k_μ.

When this algorithm ends, the topographic map defined by the level sets in $\mathcal{P}$ is a morphological segmentation of u so that each ∂P_i is made by pieces of morphological edges : $\mathcal{P} = \{P_i\}_i = \{L^k_\lambda\}_{k,\lambda} \cup \{M^k_\mu\}_{k,\mu}$. The resolution of the segmentation that is, the visual significance of the less perceptive edge, is given by the scale parameter s : when s tends to 0 almost all level sets are mapped (even those which are not perceptually significant), and when s tends to 1 only level sets with borders matching the most important perceptual edges are considered. Example of a segmentation at two different scales is shown Figure 3, at the first column of the second and third lines.

Remark

- The scale is not directly related to the size of the regions. Although level sets of big size are likely to contain more T-junctions than level sets of smaller size, the level lines corresponding to a border of a small object may be recorded at a coarse scale, while the level lines corresponding to the gradation of the light inside a large surface may not.
- The algorithm works for natural images (i.e. photographs of the real world) only. For synthetic images, one would like to replace the T-junction criterion by something related to the contrast of the level lines. With natural images, we should not use the contrast, since the segmentation would not be morphological.

3 Compact and multiscale image model

The compact image model is based on the multiscale segmentation and on pixel's values needed to reconstruct an approximation of the original image.

We use a piecewise-smooth and non-oscillating approximation v_s of the image u in each region defined by the multiscale segmentation. In this way, we remove the upper part of the total variation of u. In [13], we show how the total variation can be related to perceptual microtextures : microtextures correspond to fast oscillating parts at every scales, and therefore are associated to high variations. On the other hand, edges surrounding flat regions generate little variations. The coarea formula [11] allows to link the total variation of u with the total length of its level lines :

$$TV(u) = \int_{\mathbb{R}} \mathrm{per}(L_\lambda)\, d\lambda. \tag{5}$$

Therefore, our morphological approach leads us to split the information between edges and microtextures. More precisely, the compact image model tends to remove the microtextures when the scale s increases, whereas borders of the main structures are kept. For this reason, the approximation v_s is a sketch of u at the scale s of the segmentation. One may also want to conserve the microtexture and to code them apart. This can be done by computing the error image $w_{1-s} = u - v_s$ and by removing some residual sketch information in w_{1-s}. This problem is developed in [13] and will not be addressed here.

Our model contains two types of data : geometrical data record pixel locations on the grid, and numerical data are related to the gray level values at these locations. The numerical data are used to compute the sketch v_s on u defined on each region P_i of the segmentation. The multiscale segmentation algorithm ensures that no important edge can be located inside P_i : this explains why the image may be well approximated by a piecewise-smooth function v_s. In addition, the approximating function is chosen non-oscillating so that it catches the sketch and not the textures. How should be chosen the data to allow a good approximation ? Since the border of P_i is made by morphological edges, the knowledge of u in the internal and external side of each edge is the basic information. In order to get a compact model, we propose to retain only two samples of u for each level line (one for the internal side and one for the external). Since the external side corresponds to the internal side of the adjacent level line (remember that both lower and upper level sets are recorded), it is actually enough to retain one value per level line.

A pixel x of ∂P_i may be associated to several morphological edges (it may belong to different level sets). In that case, $v_s(x)$ is set to be the closest value to $u(x)$. This operation corresponds to chose the smallest level set, and it is implemented by means of an "inf" (lower level sets) or a "sup" (upper level sets) on all level lines containing x. In the internal side of a level line ∂P_i, we have to chose how to represent a sequence of gray level values by only one. The values may be chosen to lower the variation of v_s between two neighbouring regions of a T-junction. This not only helps to get a non-oscillating function, but also prevents the appearance of visual artefacts near the edges by keeping a low contrast. In that case, the value is the "sup" of the gray levels (lower level sets) or the "inf" (upper level sets). Another possibility is to try to preserve the average gray level of the border by computing the median or the mean value.

These remarks lead the following algorithm which computes the values $\phi_i(x)$ of v_s on each ∂P_i :

Algorithm to get samples of the approximating function -

Step 1 : Compute v_s *on internal boundary associated to lower level sets, as follows.* $\forall P_i \in \mathcal{P}, \forall x \in \partial P_i$, *if* $\exists k, \lambda / x \in \partial L^k_\lambda$, *and if* $\phi_i(x)$ *has not been already defined, define*

$$\phi_i(x) = \inf_{\lambda, k / x \in \partial L^k_\lambda} \sup_{y \in \partial L^k_\lambda} u(y). \tag{6}$$

Step 2 : Compute v_s *on internal borders associated to upper level sets, as follows.* $\forall P_i \in \mathcal{P}, \forall x \in \partial P_i$, *if* $\exists k, \mu / x \in \partial M^k_\mu$, *and if* $\phi_i(x)$ *has not been already defined, define*

$$\phi_i(x) = \sup_{\mu, k / x \in \partial M^k_\mu} \inf_{y \in \partial M^k_\mu} u(y). \tag{7}$$

To summarize our discussion, the compact image model contains the following data :

- the segmentation map $(\partial P_i)_i$;
- the sequence of samples $(\phi_i(x))_{i; x \in \partial P_i}$.

Example of such data is shown Figure 3, at the second column of the second and third lines.

The issue of computing an approximating function v_s from the samples $(\phi_i(x))_{i; x \in \partial P_i}$ belongs to the class of interpolation problems. Different approaches using image interpolation techniques have been described in the literature (see for example [17],[4],[12]), some of them including the same motives than ours of catching the image sketch in a compact way. Recently, a morphological interpolation technique for image coding has been proposed by J.Casas in [5]. In our knowledge, our work is the first one to use a segmentation map not based on a classical (and non morphological) edge detector, but on a selection of the level sets that carry the atoms of the perception.

We shall retain the work of V.Caselles, B.Coll and J.M.Morel in [7], where they extend the Casas' morphological interpolation technique, and where they prove that any interpolation operator (satisfying fairly conditions, such as morphological, invariance and regularity properties) comes down to let evolve the interpolating function r_i on each P_i with the following equation :

$$\begin{cases} \dfrac{\partial w}{\partial t} = D^2 w\left(\dfrac{Dw}{|Dw|}, \dfrac{Dw}{|Dw|}\right) & \forall t>0,\ \forall x \in P_i; \\ w(0,x) = w_0(x) & \forall x \in P_i; \\ w(t,x) = w_0(x) = \phi_i(x) & \forall t > 0,\ \forall x \in \partial P_i. \end{cases} \tag{8}$$

We have written by Dw the gradient of w along the spatial coordinates, and by D^2w the Hessian of w, that is, the matrix of the second derivatives of w.

Under some reasonable conditions [3], there exists a unique continuous viscosity solution $w(t,x)$ of (8) such that $w(t,.)$ is a Lipshitz function for all $t>0$ on each P_i, with uniformly bounded Lipschitz norm. When $t \to +\infty$, $w(t,.) \to r_i$ with $r_{i|\partial P_i} = \phi_i$. The function r_i is an absolutely minimizing Lipschitz interpolant of ϕ_i inside P_i, or AMLE for short. The evolution equation (8) can be solved using an implicit Euler scheme, so that to transform the evolution problem to a sequence of non linear elliptic problems, which leads in a discrete case to an implicit difference scheme. The sketch v_s is defined by $v_s = r_i$ on each region P_i of the multiscale segmentation $\mathcal{P}$.

Figures 3 and 4 give examples of a sketch image at different scales.

4 Application to image compression

In order to show how the multiscale image model is compact, the geometrical and numerical data have been error-free compressed using adapted coding techniques. We do not claim that our compression scheme gives better results than well established compression standards, further developments are needed before we could present a fair comparison. We just wish to mention the main advantage of a model based on level sets : the segmentation is not only invariant by affine map and by contrast change, but it permits to code very precisely the perceptual edges. On the contrary, compression schemes based on a space-frequency or on a linear scale-space representation cannot respect the perceptual edges when the compression ratio is to high : blocking artifact called Gibbs phenomena appears in the neighborhood of the edges, due to the quantization of the Fourier coefficients which have a bad decay at discontinuities. The same phenomena occurs with wavelet coefficients, althought their space-localization reduces the artifacts.

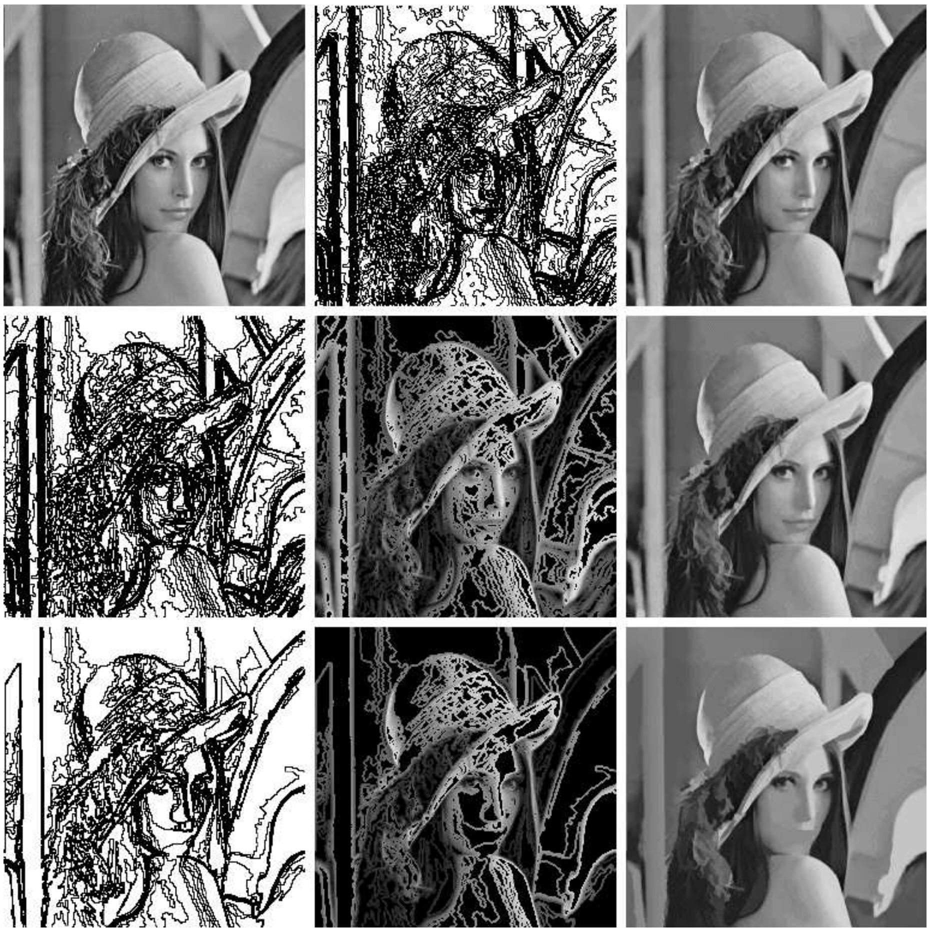

Fig. 3. This plate illustrates the multiscale image model. First line, from left to right: Original Lenna 256×256 image; a topographic map which gives the borders of the lower level sets L_λ for $\lambda = 8n, n = 0, \ldots, 32$; reconstruction using the lower level sets of the previous image only. Second line, from left to right: segmentation at scale $s = 0.65$; samples $\phi_i(x)$ used to compute the sketch; sketch at scale $s = 0.65$. Third line, from left to right: segmentation at scale $s = 0.90$; samples $\phi_i(x)$ used to compute the sketch; sketch at scale $s = 0.90$.

Figure 5 illustrates this problem. We have compressed the "House" (a simple image with strong edges) at a high compression ratio of 38. Both JPEG [24] (the standard for Fourier-based compression) and the Shapiro's EZW [23] (Embedded Zerotrees Wavelet, one of the best wavelet-based compression scheme) algorithms give very poor results. Our compact image model allows to code the "House" at the same compression ratio without distortion of the edges. Of course, only the most important structures are preserved. To keep some texture, we propose

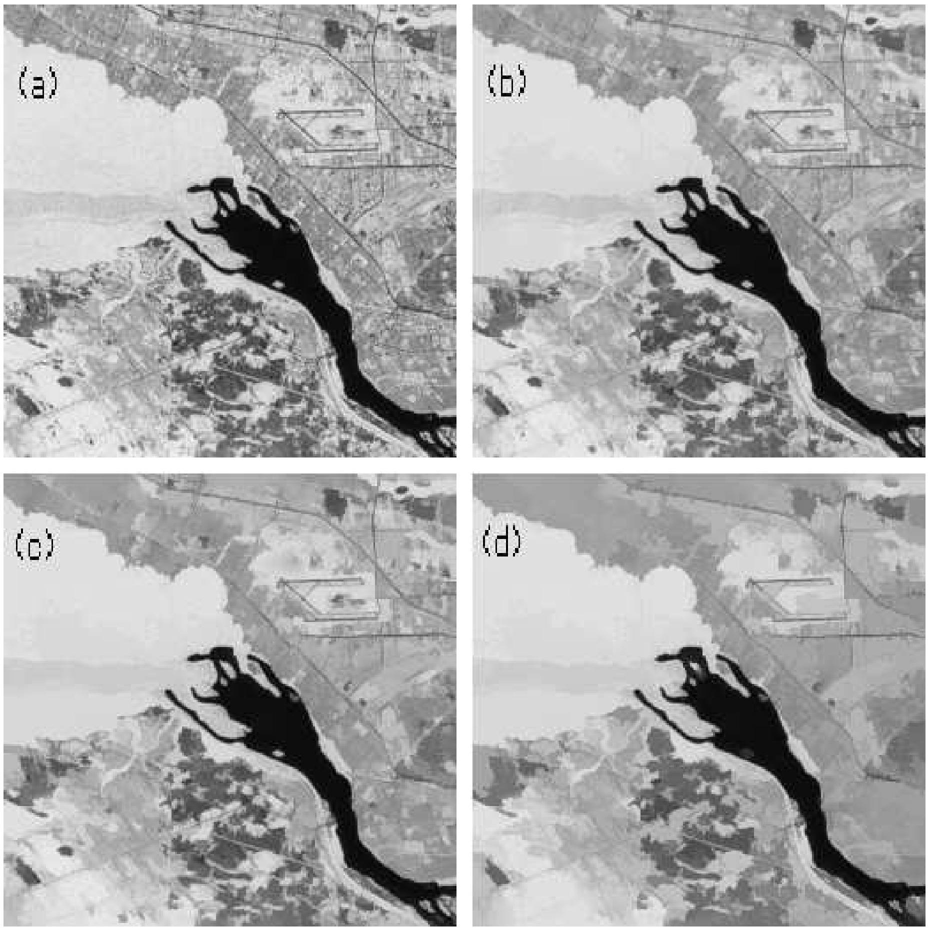

Fig. 4. Multiscale analysis of a view from a satellite. (a): original 256×256 image. (b): sketch at scale $s = 0.90$. (c): sketch at scale $s = 0.96$. (d): sketch at scale $s = 0.99$. Only 1% of all lower level sets and 1% of all upper level sets are used to reconstruct this image.

in [13] to compute the error image and to compress it using a linear scale-space representation : since the error image does not have to code edges, we do not introduce blocking artifact.

5 Conclusion

The level sets decomposition is a well-known tool to get an invariant and morphological image representation. But it is perceived as a redundant structure generating an huge quantity of data. In this paper, we propose a method to select the most representative level sets. From these level sets, a sketch image

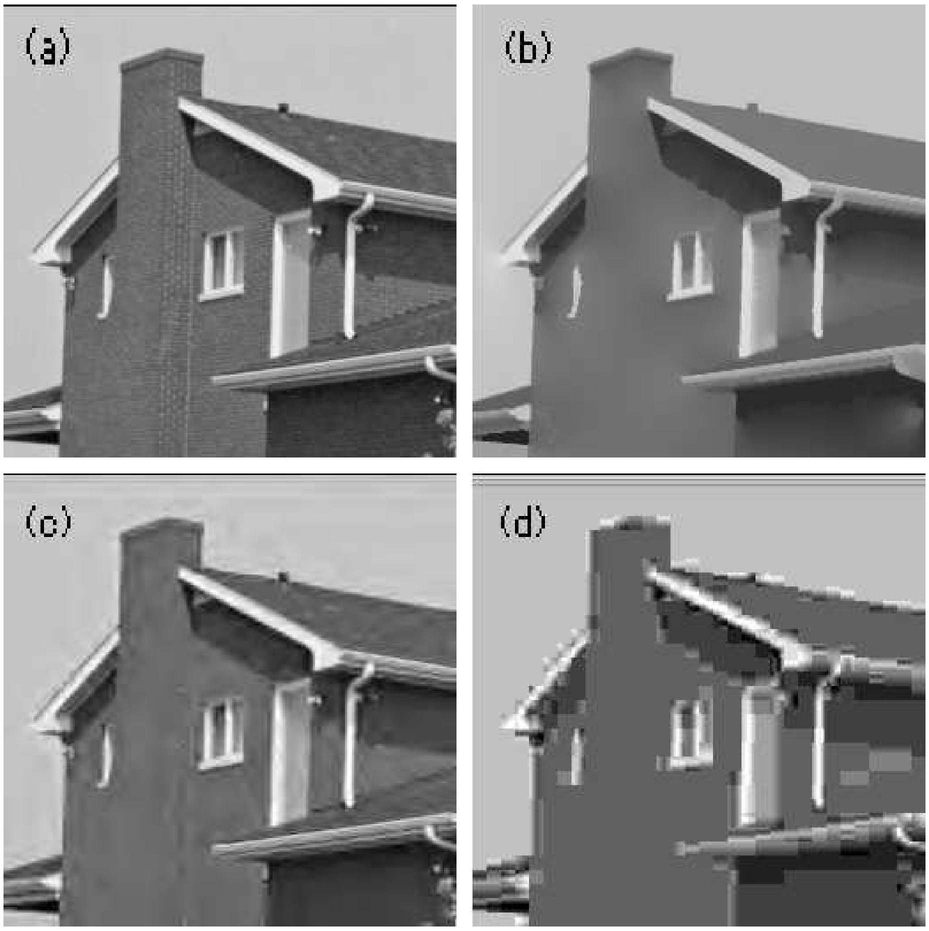

Fig. 5.
Comparison with linear scale-space and space-frequency compression schemes. (a): original "House" image. Size 256×256, 8 bpp (bit per pixel). (b): Compact image model of (a) at a scale that leads a bit rate of 0.21 bpp. This bit rate corresponds to a compression ratio of 38. (c): Image (a) compressed at the same ratio using a biorthonormal wavelet scheme (EZW). (d): Image (a) compressed at the same ratio using a windowed Fourier scheme (JPEG).

can be reconstructed. It contains the borders of the most important structure, up to a scale parameter. This requires a new definition of edges, as level lines containing a great number of topological singularities (the T-junctions). This approach allows to build a non-linear scale-space image model respectful of the human visual system, and with a compactness that makes it suitable to perform image compression at low bit rate.

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