# Perceptible Level Lines and Isoperimetric Ratio

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Jacques Froment
PRISME, UFR Mathématiques et Informatique
Université Paris 5 R. Descartes
45 rue des Saints-Pères, 75270 Paris cedex 06, France.
and
CMLA, Ecole Normale Supérieure de Cachan,
61 avenue du Président Wilson, 94235 Cachan cedex, France.

## Abstract

*This paper introduces a simple criterion to select the most important level lines from the numerous set obtained with a topographic map. A topographic map gives a morphological and a geometrical representation of the information contained in natural images. Most of level lines are perceptually negligible and may be removed without noticeable distortion. Such curves, which do not carry information on the shape of the objects, are oscillating very quickly. They can be discriminated from the others by means of the isoperimetric ratio of the region they delimit. The main advantage of the isoperimetric criterion lies in its invariance with respect to contrast changes. As a result of, the topographic map of the remaining level lines achieves a segmentation of the image with most borders matching the perceptual edges. This segmentation being invariant under changes of contrast is a great alternative to classical segmentation or edge detection methods, when to be used to compare the content of two images, such as in stereo matching or motion analysis applications.*

## 1. Introduction and Background

The problem of constructing contrast invariant edge-based representation of images should be an important issue of Image Processing. Number of Computer Vision algorithms are based on the comparison of edges between two images. If the necessary condition of a representation invariant under translation has been acknowledged in the past, the significance of the contrast invariant property has been pointed out recently [3] : the light captors of cameras are nonlinear increasing functions, and these functions differ from one camera to the other (the response of a given camera is even changing with time). In addition, lighting condition in an outdoor scene is continually changing, generating changes of contrast. An indoor scene may also be concerned by such modification : a moving object or a moving camera with exposure set to automatic will experience contrast changes.

Topographic map of gray-level images has been introduced in this framework [3], as a geometrical and contrast invariant representation of the information contained in natural images. First applications of topographic maps include extraction of shapes [9, 6], comparison of images [1, 9], structured compression [5] and disocclusion [8] (recovery of hidden parts of an object occluded by another one). A lower (or upper) topographic map of a gray-level image $u : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ is a subset of the connected components of the lower (or upper) level sets of $u$, a lower level set $[u \leq \lambda]$ being the set of pixels $x$ such that $u(x)$ is lower than $\lambda$. When all connected components are considered and if the family of levels $(\lambda)$ associated to the connected components is recorded, the original image $u$ may be reconstructed *i.e.*, the topographic map yields to a complete representation :

$$u(x) = \inf_\lambda [u \leq \lambda] = \sup_\lambda [u \geq \lambda]. \tag{1}$$

Let $g$ be a contrast change, that is an increasing continuous real function. Since we have

$$[g(u) \leq \lambda] = [u \leq g^{-1}(\lambda)], \tag{2}$$

the family of level sets of $g(u)$ is equal to the family of level sets of $u$ : a contrast change preserves the topographic map. In [3] it is proved a stronger property : the topographic map of an image $u$ is equal to the topographic map of another image $v$ if and only if there exists a function $g(x, \lambda)$ increasing in $\lambda$ such that $v(x) = g(x, u(x))$ for all pixels $x$ (there are other technical properties on the function $g$, see [3] for details). It means that $v$ is obtained from $u$ by applying a local contrast change. Topographic maps are therefore equivalence classes of images, modulo local contrast changes.

If $u$ is of bounded variation, the connected components of level sets can be characterized by their boundaries, which are unions of Jordan curves, the curves being closed or touching the image border $\partial\Omega$. Those curves are called the level lines of $u$. Since perceptual edges refer to discontinuity lines in $u$, lot of level lines go through an edge location. Therefore, a topographic map tends to give prominence to the perceptual edges. However, a great number of level lines do not reveal any edges. The issue we address is the following : how could we discriminate the perceptible level lines, which are associated to edges, from the negligible ones ? Various interesting answers have been already presented (see for example [11, 7]), but none of them preserves the contrast invariance property. In [3, 5], the number of junctions a level line contains is used to characterized its perceptual significance. This topological criterion is contrast invariant in the continuous framework, but not in the discrete one : the algorithm described in [3] needs a quantization threshold to detect the true junctions from junctions due to noise.

## 2. Isoperimetric ratio of Shapes

The algorithm we propose is based on the following remark : in a uniform region, when a level line is not associated to a discontinuity line, its geometry is essentially settled by the noise. As a result of, such level line is continually oscillating (see the image C in Figure 1). Oscillations are related to the total variation of $u$, and not to the regularity of the curve.

Let $O$ be a bounded region of $\mathbb{R}^2$ and $a(O)$ its area, which is the two-dimensionnal Lebesgue measure of $O$. We suppose that the boundary of $O$ is a countable union of rectifiable curves, so that its length $l(\partial O)$ is well defined. We define the *isoperimetric ratio* of $O$ to be the number

$$i(O) = \frac{l^2(\partial O)}{a(O)}. \tag{3}$$

The isoperimetric inequality [10] ensures that $i(O) \geq 4\pi$, and we have $i(O) = 4\pi$ if and only if $O$ is a disk. For a rectangle, we have $i(O) = 16$. Let us consider such a region made by a non-oscillating boundary. Now let $O'$ be a distorted version of $O$, obtained by adding oscillations to the boundary. Since $l(\partial O') \gg l(\partial O)$ and $a(O') \simeq a(O)$, we get $i(O') \gg i(O)$. This shows that the isoperimetric ratio is increasing accordingly to the boundary's oscillations. We propose to label a level line as negligible if it is a boundary of a region $O$ with large isoperimetric ratio.

A level line is part of a boundary of a connected component $C$, but since $C$ may be not simply connected, it may contains holes. A level line in the interior of $C$ (the boundary of an hole) may be oscillating whereas the level line of the exterior may not. It is therefore not very meaningful to consider $i(C)$. We will rather examine $i(S)$ where $S$ is the *shape* associated to a unique level line $L$ that is, the interior region delimited by the closed curve $L$ (if $L$ is encountering the image border $\partial\Omega$, it is not closed. We close it by adding the minimal path contained in $\partial\Omega$).

The algorithm is straightforward. A fast algorithm to compute shapes is described in [9]. To avoid meaningless detection, shapes below a given size are not recorded. For each remaining shape $S$ we compute the isoperimetric ratio $i(S)$. In the discrete case, $a(S)$ is simply the number of pixels in $S$ while $l(S)$ is the number of pixels of its level line. The shapes are sorted in ascending order according to their isoperimetric ratio. Experiments are summarized by the figure 1. We have compared our result with the well-known Canny-Deriche's edge detector [2, 4], which is not contrast invariant as the images G,H,I prove it. Our algorithm achieves a perfect contrast invariant detection : the images J,K,L are exactly the same whether they have been computed from E or from F. We notice in J,K,L that some level lines have been mislabeled, since they do not appear to be associated to a perceptual edge while their shape has a low isoperimetric ratio. This may happen when a level line is both part of an edge and of a uniform region.

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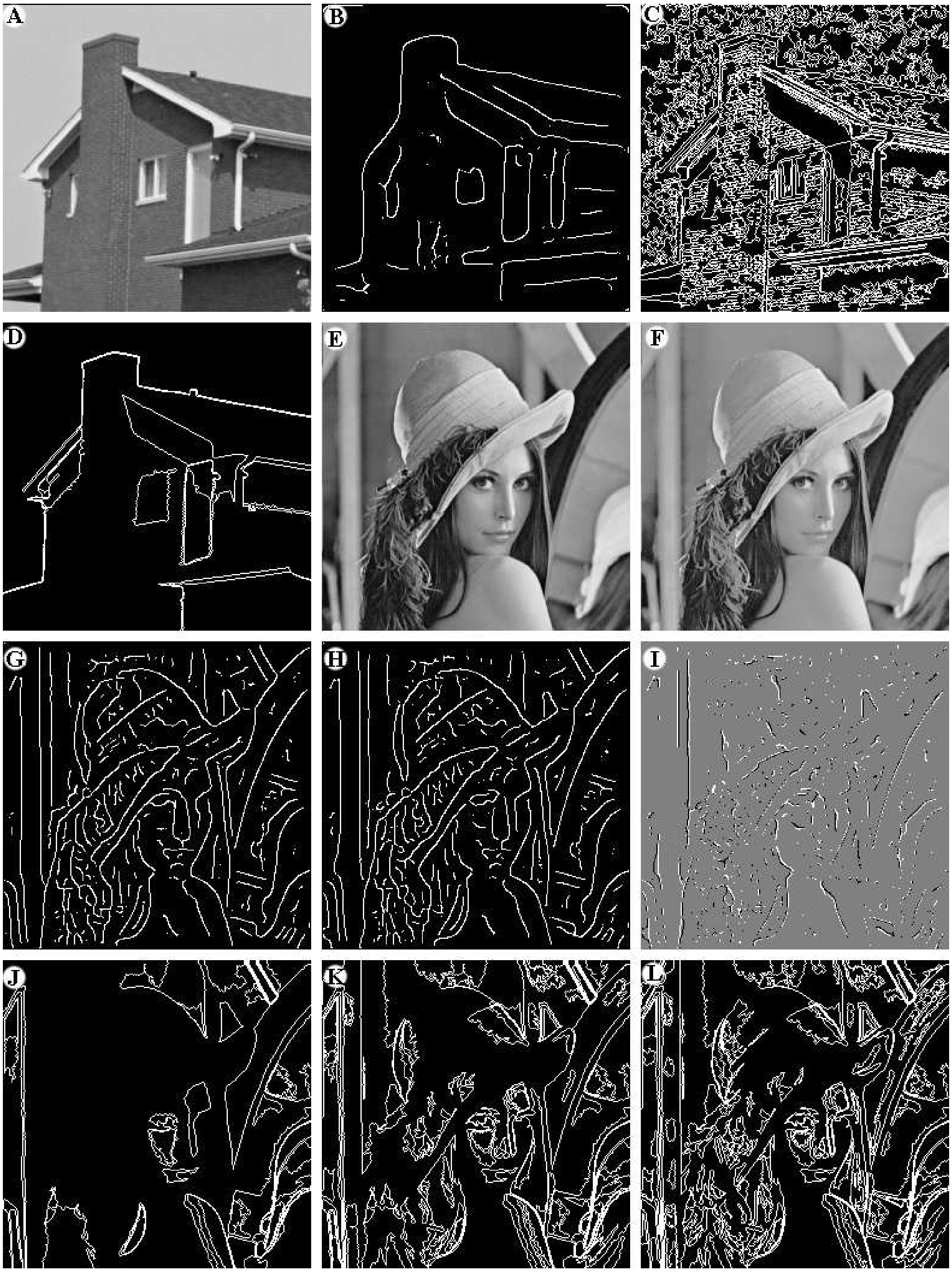


**Figure 1. A : “House” image. B : Canny-Deriche’s edge detector applied on A. C : Topographic map of A, with shapes of high isoperimetric ratio (range from $100$ to $14000$). D : Topographic map of A obtained using our algorithm (first $11$ shapes, isoperimetric ratio from $25$ to $80$). E : “Lena” image. F : E with a change of contrast. G : Canny-Deriche’s edge detector applied on E. H : Canny-Deriche’s edge detector applied on F. I : Difference $G - H$. J,K,L : Topographic maps of E (or F) obtained using our algorithm (J : first $44$ shapes, isoperimetric ratio from $27$ to $73$; K : $112$, $27$ to $109$; L : $186$, $27$ to $130$).**