Presentation of the thesis topic
DNIVARIAD
Deep Neural Networks Integration in Variational Methods for Image Restoration: Theoretical and Algorithmic Development

Context and challenges of the project

The research project, spearheaded by Jacques Froment's team specializing in mathematical image processing at LMBA-UBS (UMR CNRS 6205), focuses on modeling relevant information present in digital images. Their primary interest lies in leveraging these models to tackle complex inverse problems concerning image restoration and reconstruction. The recent successes of deep learning in image processing have prompted a reconsideration of the mathematical approaches that have traditionally guided the development of image processing algorithms for decades. In collaboration with partners like Alexandre Bousse (LaTIM UMR Inserm 1101, Brest) and Franck Vermet (LMBA-UBO), the team works on integrating deep neural networks into variational approaches. This integration aims to enhance methods for reconstructing medical images, particularly those from new spectral scanners (spectral tomography). The expertise acquired through two co-supervised theses in ongoing projects [1]–[4] opens the way to new avenues of research, aiming to generalize the approaches developed in medical imaging contexts. The objective of this new thesis is to propose a suitable formal framework adaptable to various ill-posed inverse problems in image reconstruction, and to apply it to new domains such as satellite imagery.

The TAMI research team, led by Bartomeu Coll Vicens at the University of the Balearic Islands (UIB, Spain), focuses on mathematical image and video processing and analysis linked to applications in satellite imaging, medical imaging, and digital photography applications. Previous collaborations between the teams of Jacques Froment and Bartomeu Coll have resulted in innovative algorithms related to satellite imagery, specifically in the fusion of hyperspectral and multispectral images [5], [6].

Over the last decade, the advent of deep learning applied to computer vision and then to image processing has considerably changed the landscape of mathematical image processing [7]. However, despite the remarkable performance provided by deep neural networks, they fail in certain scenarios and are susceptible to adversarial attacks, raising concerns about their stability, interpretability, and explainability. Despite recent advances in interpretability, deep neural networks still retain a “black box” aspect due to their complex architectures and multitude of parameters. This lack of interpretability poses challenges for understanding and explaining their decision-making processes, contrasting with the transparency of mathematical models used in image processing. Effectively integrating mathematical models with deep learning to leverage the strengths of both approaches while maintaining interpretability and understanding remains a significant challenge.

The project aims to address these challenges by proposing a general theoretical framework and adapting it to the resolution of different inverse problems, particularly in the field of satellite imaging and/or in that of medical imaging.

Methodological and technical approaches

In addressing the challenges of stability, interpretability, and explainability when using deep neural networks for image processing tasks, a solution brought by Jacques Froment's team involves viewing the neural network's black box as a more detailed method for representing visual information, supplementing the previously established mathematical models. The focus is on integrating this black box element into the current mathematical models. Initial solutions were suggested, including the incorporation of generative models to regulate energy functionals such as in [1], [2] or to learn a prior probability distribution to invert a stochastic differential diffusion equation such as in [3], [4]. Another approach involves integrating neural networks into iterative methods to extend the computations outlined by variational methods. This method, known as algorithm unrolling [8], is particularly employed within Bartomeu Coll's team to address the challenge of satellite images fusion [9].

In the application framework of satellite images fusion, it is proposed to modify the variational approaches developed by Bartomeu Coll's team in order to include a generative model. Images fusion seeks to leverage well-resolved spatial imagery combined with low spectral resolution (multispectral or panchromatic images) and less spatially resolved yet high spectral resolution (hyperspectral images) to create a single image with optimal spatial and spectral resolutions. A solution could be to add a regularization term forcing the images at different spatial and spectral resolutions to be close to the output of generative U-nets from the same reference image, which would then be the fused image at high spatial and spectral resolution.

An alternative would be to define a probabilistic generative model: the training data is progressively degraded by noise whose variance is slowly increased and the training of the neural network consists of reversing the degradation in order to obtain a generative model of this data. It appears possible to inject a data attachment term to such a generative model, in order to ensure the generation of the image belonging to the variety generated by the images of the training data which is as close as possible to the data observed (images at different spatial and spectral resolutions within this application). A variant of a probabilistic generative model would be to invert a stochastic differential diffusion equation over time by estimating the score, i.e. the gradient of the log-probability of the density associated with the data.

These different approaches being versatile, it would also be possible to combine them within a single energy functional and, in terms of algorithm unrolling, in the associated iterative method. Specifically, energy functionals found in imaging inverse problems consist of multiple terms combined, such as data attachment and regularization, which are convex functions not always differentiable. The concept of a proximity operator, extending the convex projection operator, enables the expression of the optimization problem as a fixed-point problem, resulting in the creation of forward-backward splitting algorithms [10]. It has recently been established that using algorithmic unfolding to encode the proximity operator may significantly enhance the performance of these algorithms [11], [12].

The study's objective is twofold: to achieve results that not only demonstrate superiority compared to the current state of the art but also to enhance the existing mathematical framework. Specifically, this involves defining anticipated performance criteria and suggesting methodologies that can be universally applied to numerous imaging inverse problems.

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