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Mathematical modelling of coastal systems: engineering approaches for parameter identification, validation and analysis of the models.

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Introduction

Coastal zones are complex systems characterized by large fluctuations in the physical and chemical parameters. This is primarily due to their location between land and open sea, which makes their equilibrium strongly influenced by the quality of inland waters flowing into them, rich in organic and mineral nutrients derived from urban, agricultural and/or industrial effluents and domestic sewage. In addition, costal systems are in many cases subject to strong anthropogenic pressures due to tourism and/or intensive shellfish/fish farming. Related to this, multiple and uncoordinated land-use modifications such as structural changes in lagoon topography, artificial increase of the number of sea connections, or changes in bathymetry, are often undertaken with limited sectorial objectives in mind. All these factors are responsible for important ecosystem alterations characterised by eutrophic/dystrophic conditions, algal blooms, oxygen depletion and hydrogen sulphide production. Additional problems arise from cost erosion, subsidence and effects related to extreme meteorological events.

In view of the above discussion, the development of management and control tools for coastal zones is a complex task requiring interdisciplinary research and active interaction with the end-users. Furthermore, the application of mathematical models becomes a fundamental step for handling and reducing the complexity of these ecosystems.

The problem of modeling coastal ecosystems can be faced with several approaches. In this report modeling stage, identification and qualitative analysis of the models as well as integration with socio economic and multicriteria analysis are presented.

1 Modeling stage

Mathematical models of biological, physical and chemical processes are fundamental tools for analyzing disruptions in the ecosystem due to abnormal conditions (see, e.g., [1, 3, 34, 10] and references therein for some examples related to Mediterranean lagoons).

A nonlinear model applied by the author for the study of the Lagoon of Caprolace (Italy) is here considered. The model accounts for two integrated compartments: the first compartment describes the phytoplankton-zooplankton dynamics and is integrated with a second one accounting for the oxygen-nutrients dynamics. Several exogenous inputs are also considered, such as light, wind and temperature. The model is described in the following.

1.1 Driving environmental functions

The exogenous inputs to the lagoon system described in this subsection are light, wind and temperature. Such inputs usually show an annual periodicity, inducing seasonal phenomena in ecosystems [15]. The month is assumed as sampling time for our analysis.

Based on references [4, 5, 11, 15, 30], we assume the following equations for describing the annual variation of the photoperiod $u_1(t)$, the temperature $u_2(t)$ and the average wind intensity $u_3(t)$:

$$u_1(t) = \mu_{11} + \mu_{12} \sin\left(\frac{2\pi}{12}(t + \mu_{13})\right), \tag{1}$$

$$u_2(t) = \mu_{21} + \mu_{22} \sin\left(\frac{2\pi}{12}(t + \mu_{23})\right), \tag{2}$$

$$u_3(t) = \mu_{31} + \mu_{32} \cos\left(\frac{2\pi}{12}(t + \mu_{33})\right), \tag{3}$$

where $\mu_{ij},\ i, j = 1, 2, 3$ are parameters to be fixed on the basis of the available data.

An additional variable playing an important role in the model is the limiting factor of temperature on the growth of phytoplankton biomass. According to the analysis proposed in [15], this limiting factor u_4 can be expressed by the equation $u_4(t) = \gamma e^{\delta u_2(t)}$.

1.2 State variable equations

In this subsection, the dynamic state equations representing the mathematical model will be introduced. The model parameters, denoted by k_i, $i = 0, 1, \ldots,\ 16$, k_P, k_T, k_X, k_{AE} and k_{AN}, are all non negative.
Phytoplankton. This population represents the producer (in forms of Diatoms, Peridenes and Microflagelates), i.e. the set of vegetal species performing carbon fixation, measured in terms of biomass. The dynamic equation describing the evolution of the phytoplankton x_1 is

$$\dot{x}_1 = k_1 u_1 u_4 x_1 \frac{x_4}{k_0 + x_4} - k_2 x_1^2 - k_3 x_2 \frac{x_1}{k_P + x_1}. \tag{4}$$

The first term in equation (4) accounts for the photosynthetic activity, which produces oxygen leading to an increase of phytoplankton biomass. The process is influenced by light intensity and limited by temperature and nutrient concentration. The second term of the equation represents the natural mortality that is assumed to be proportional to the square of phytoplankton biomass itself. The formulation of the first two terms is based on the logistic equation [20], which is the best known model of population dynamics for the vegetation microorganisms. The third term accounts for losses due to grazing of zooplankton.

Note that k_P represents the half saturation value, since if $x_1 = k_P$, then $x_1/(k_P + x_1) = 0.5$, thus producing a reduction of 50% of the grazing and growth of zooplankton (see equation (5) below).
Zooplankton. The variable x_2 is the biomass of herbivore consumers (mainly copepods as some species of Acartia). The zooplankton evolution equation is

$$\dot{x}_2 = k_4 x_2 \frac{x_1}{(k_P + x_1)} - k_5 x_2. \tag{5}$$

The dynamics of this variable is regulated by a growth due to grazing on phytoplankton and by the losses for natural mortality. Here, it is assumed that there is not interspecific competition between zooplankters, as it can be checked in the mortality term.
Dissolved oxygen. The equation describing the dynamics of dissolved oxygen x_3 is

$$\dot{x}_3 = k_6 u_3 + k_7 u_1 u_4 x_1 \frac{x_4}{k_0 + x_4} - k_8 f_\alpha(x_3) - k_9 f_\beta(x_3) + k_{10} \frac{[C_S(u_2) - x_3]}{k_T + u_2} - k_{11} x_1 x_3 - k_{12} x_2 x_3, \tag{6}$$

where

$$f_\alpha(x_3) = \frac{x_3^2}{k_{AE} + x_3^2} \qquad f_\beta(x_3) = \frac{x_3}{k_{AN} + x_3^2}. \tag{7}$$

The first two terms of equation (6) account for wind re-aeration and photosynthesis oxygen production. The third and fourth terms represent the losses due to the bacterial activity, while the last two terms show the decrease of oxygen content due to the phytoplankton and zooplankton respiration. The fifth term accounts for the equilibrium physical-chemical reaction between gaseous oxygen and dissolved oxygen. The rate of this process is globally influenced by water temperature: the higher the temperature, the lower is the probability that the exchange occurs from air to water. The parameter k_T represents the temperature limiting factor. Furthermore, since the rate k_{10} is always positive, the direction of the chemical reaction $[C_S(u_2) - x_3]$ is related to the over/under saturated level of oxygen concentration x_3 in the water. For temperature ranges between 7 and 28 $^\circ C$, the saturation value $C_S(u_2)$ changes according to the equation $C_S(u_2) = c_1 + c_2\ u_2 + c_3\ u_2^2$, [30].

Finally, the functions f_α and f_β, described in equation (7), represent the dependence of aerobic and anaerobic activities on dissolved oxygen concentration, respectively. Indeed, the degradation of organic matter performed by anaerobic bacterial pool (in water and sediments) is a very important process responsible for about 50% of the mineralization of organic matter [12]. This process always occurs under environmental conditions that cause the amount of dissolved oxygen to be insufficient for complete mineralization. If the matter undergoes anaerobic breakdown, the result is fermentation leading to sulphate reduction [13]. Bacterial sulphate reduction indirectly consumes the oxygen available in the water column, since the hydrogen sulfide produced as a reaction to anaerobic respiration is re-oxidized chemically using a double amount of oxygen compared to the requirements of the aerobic process, thus exacerbating the oxygen shortage.

Note that k_{AE} and k_{AN} define the shape of f_α and f_β, respectively, and they will be fixed on the basis of availability of oxygen in the considered lagoon.

Nutrients. The nutrient variable x_4 refers to the nitrogen and phosphorus compounds in water and sediments. There is evidence from field experiments that in coastal shallow water lagoons the main source of nutrients for phytoplankton growth must come from recycling due to bacterial activity and sediment release, while the losses are due to the photosynthetic activity, outgoing flows of water and material and retaining from sediment. We model nutrients dynamics according to the following equation:

$$\dot{x}_4 = \ k_{13} f_\alpha(x_3) + k_{14} f_\beta(x_3) - k_{15} u_1 u_4 x_1 \frac{x_4}{k_0 + x_4} - k_{16} \frac{x_4}{k_X + x_4}, \tag{8}$$

where $f_\alpha(x_3)$ and $f_\beta(x_3)$ are given in (7) and k_X represents the nutrient fixation half saturation constant.

The first two terms of equation (8) account for the aerobic and anaerobic production of nutrients by mineralization of organic matter, respectively. The consumption terms are due to the photosynthetic activity of phytoplankton species and to the fixation in the sediment.

2 Identification of the models

Parametric identification deals with the problem of estimating mathematical models for the description of dynamic systems. For a thorough treatment of the subject the interested reader is reffered to the book [18].

The word *identification* denotes the set of techniques which permit to build mathematical model of dynamic systems starting from experimental data.

The main elements of the problem are:

- a data set; for example a sequence of N samples of the input $u(t)$ and the output $y(t)$ of the dynamic system, i.e.:
$$\mathcal{Z}^N = \{u(1), y(1), \ldots, u(N), y(N)\}; \tag{9}$$
- a class of candidate models (for example: transfer functions of fixed order);
- a selection criterion for choosing the "best" model within the considered model class;
- model validation procedures.

The elements above are detailed in the following.

Data set

The *data* are the set of measurements collected from the system, that are used to estimate a model describing the system itself. We consider only input-output data in the time domain, such as $\mathcal{Z}^N$ in (9).

In several applications, it is possible to choose the input signal, or the operative conditions of the system. As we will see, this can deeply affect the quality of the identified model. Other important issues are the duration of the experiment (the length N of the data set), the sample time, etc. All these aspects are crucial in the design of the "identification experiment", which is the first phase of the entire procedure. Unfortunately, sometimes the selection of the input is not possible and the operative conditions cannot be changed (e.g., when the measurements are collected from a real ecosystems).

Model class

The selection of the model class represents the most delicate part of the identification procedure, because it deeply influences the final result.

In general, a model can be seen as a map from data to the output:

$$\hat{y}(t;\theta) = g\left(\theta, \mathcal{Z}^{t-1}\right)$$

where $\hat{y}(t;\theta)$ represents the output at sample time t predicted by the model, $\mathcal{Z}^{t-1}$ are the collected data until time $t-1$, and θ is a set of elements (*parameters*) which define the specific model inside the class.

The model class is then represented by:

$$\mathcal{M} = \{g(\theta, \cdot) : \theta \in \Theta \subset \mathcal{S}\}$$

where Θ is the set representing the prior knowledge on the system and $\mathcal{S}$ is the space of elements θ.

In the class of parametric models θ is a finite dimension parameter vector (e. g., the coefficients of a rational transfer function $G(z)$ or the constants of the model).
In parametric identification, we can distinguish between models in which the parameters derive from physical laws, such as ecological or biogeochemical models, or some other knowledge on the system ("grey-box" models), and models in which the parameters are used to represent the input-output behavior of the system, without any specific physical meaning ("black-box" models).

Furthermore, the models can be classified according to their complexity (linear or nonlinear, low or high order, time-invariant or time-varying, ecc.). In general, a main requirement for a class of models is the capability of representing relevant phenomena in the considered field.

"Best" model selection

Once the data are collected and the model class $\mathcal{M}$ is chosen, the identification problem consists of finding the parameter vector θ^* minimizing a cost functional $J(\theta)$:

$$\hat{\theta}^* = \arg\min_{\theta\in\Theta} J(\theta)$$

within the chosen model class. Therefore, an *optimization* problem must be solved (whose complexity tipically grows with that of the class of models and of the functional $J(\theta)$).

Model validation

Once the model $g(\theta^*, \cdot)$ has been identified, we need to *validate* it. This means that we want to verify its coherence with the given data and to the prior knowledge on the system. There are several different procedures for the validation of a model. In general, a model may not pass the validation test for different reasons:

- divergency of the optimization algorithm (numeric problem);
- wrong choice of the criterion $J(\theta)$;

- wrong choice of the model class;
- not enough data.

As previous explained, the choice of the models class is a crucial point (e.g. for Linera Time Invariant models, the choice of the model order). In practice, the identification procedure is repeated until the validation criteria are satisfied, by suitably modifying one or more elements of the whole identification problem.

The cost function to be minimized in the problem of identifying the parameters of a nonlinear model described by a set of ordinary differential equations is in general non convex. In this case the presence of local minima introduces uncertainties in the initialization of the vector of parameters.

A solution to this problem, proposed by the author in [10], is to perform qualitative analysis of the models in order to make the 'a priori' information on the system useful for the aim of selecting the initial parameters to be accounted in the optimization minimization algorithm.

In this case the identification technique is based on the minimization of the cost function $F(\theta)$, representing the mean square error between simulated and experimental data [18]

$$F(\theta) = \sum_{i=1}^{N} e^2(t_i) = \sum_{i=1}^{N} (\phi(\theta, t_i) - \overline{\phi}(t_i))^T W_i(\overline{\phi}(t_i))(\phi(\theta, t_i) - \overline{\phi}(t_i)), \tag{10}$$

where $\overline{\phi}(t_i)$ is the measurement vector at time t_i, $\phi(\theta, t_i)$ is the vector of corresponding values provided by the model at time t_i, θ is the vector of model parameters, and $W_i(\overline{\phi}(t_i))$ is a suitable weighting matrix.

Referring to the model described in Section 1 the parametric identification has been performed using three sets of data: phytoplankton ($\overline{x}_1$) and zooplankton ($\overline{x}_2$) biomasses and oxygen concentration ($\overline{x}_3$). Following the notation adopted for the cost function (10), we have

$$\begin{aligned} \theta &= (k_0, \ldots, k_{12})', \\ \overline{\phi}(t_i) &= (\overline{x}_1(t_i), \overline{x}_2(t_i), \overline{x}_3(t_i))', \\ W_i &= diag\ \{\overline{x}_j^{-2}(t_i)\}, \qquad j = 1, \ldots, 3. \end{aligned} \tag{11}$$

Note that the parameter vector θ only contains a subset of the model parameters. The parameters $k_{13}, \ldots, k_{16}$, k_P, k_T, k_X, k_{AE}, k_{AN} are assumed to be known constants. This choice is suggested by the lack of data on nutrients and the knowledge of typical admissible values for some of these parameters from the literature. For example, the values of k_{AE} and k_{AN} in (7) are based on the observation that the activation/deactivation of aerobic and anaerobic bacterial activity falls in the range $[3 \div 4]\ mgl^{-1}$ oxygen.

3 Qualitative analysis

Qualitative analysis of ecological systems is a powerful tool for the investigation and prediction of the behavior of systems subject to exogenous inputs (see, e.g., [17, 16, 28]). It can be particularly useful when nonlinear models are under development, like in the case of ecological systems.

The investigation of the dynamics of the periodically forced nonlinear model (4)-(8) is of fundamental importance to understand the ecological consequences on the lagoon system behaviour. Indeed, to achieve a complete understanding of the long term behaviour of the model, a qualitative analysis is necessary to determine the existing dynamical solutions (equilibria, periodic solutions, ...) and their stability properties.

This section is devoted to this qualitative analysis, whose aim is to gain knowledge on the behavior of systems whose parameters are different from the estimated ones. Such an investigation of "nearby" systems is motivated by two main reasons. The first one is related to the uncertainty affecting the numerical parameters in the expression of the exogenous inputs, which have been estimated from the available data. The second one concerns the case when there is evidence that the identified model corresponds to an unsatisfactory local minimum of the cost function (10). In this case, qualitative analysis of nearby systems may help in selecting the most appropriate values of the parameters. This

fact is particularly useful in view of the large number of parameters of the model to be estimated and the limited amount of available real data.

The exact analysis of the dynamics of the estimated model is a prohibitive task, since such model is a fourth order nonlinear system with a large number of parameters. Therefore, a reduced order model of the dynamics based on the equations (1) and (2) and involving only two free parameters M and N is built catching the most significant biological aspects of the system For a detailed description of the reduction mechanism, refer to [10].

Qualitative analysis is performed to investigate the possible dynamical behaviours predicted by the model, as well as their stability properties as a function of the two parameters M and N. For $N = 0$, the reduced model has three equilibrium points. Two of them are trivial: the origin $(0, 0)$, which is a saddle point, and an additional saddle point E_1. The third equilibrium is the point of interest $E_2 = (x_1^*, x_2^*)$ whose explicit expression can be easily obtained from (1)-(2). The following considerations can be made on the properties of the equilibrium point E_2.

$N = 0$. The equilibrium point E_2 is stable for $M > 0.5$; for $M = 0.5$ one of the two eigenvalues vanishes; for $M < 0.5$ the model shows a saddle point.

$N > 0$. For small values of N, the periodically forced model retains the stability properties of the unforced system. In particular, the stable equilibria give rise to stable periodic solutions of period $T = 12$ months (see [7] for a theoretical explanation). As N increases, such periodic solution may undergo some bifurcation, thus generating different solutions [17, 28]. Clearly, the detection of bifurcations is important for understanding the system behaviour especially for larger values of N (see also [15]). In the following, (F) denotes a *flip* bifurcation, $(N - S)$ a *Neimark-Sacker* bifurcation and (T) a *tangent* or *fold* bifurcation.

The bifurcations can be studied via the Poincaré map [17, 28] which can be computed using suitable software packages (in our case, LOCBIF (Interactive Local Bifurcation Analyzer [14]) and WINPP (The Dynamical Systems Tool [6]) have been used). These packages perform both simulations of dynamical systems and numerical bifurcation analysis.

Figure 1 shows the complete picture in the (M, N)-plane of the bifurcations of the fixed points of the Poincaré map associated to the equilibrium studied. For any $N > 0$, the system shows at least one T-periodic solution. Its long term behaviour is clear in Figure 1, where stable T-periodic solutions are present in the grey regions only. Several bifurcation curves have been obtained. Specifically, F1 and F1-1 represent the curves of the period T flip bifurcations, T1 the fold bifurcations, and N-S1 the curve of the Neimark-Sacker bifurcations. Furthermore, F2 and F2-1 represent the curves of the period 2T flip bifurcations, T2 and T2-1 the fold bifurcations, and N-S2 the curve of the Neimark-Sacker bifurcations.

By the analysis of such bifurcation curves several conclusions on the nonlinear dynamics can be drawn. For instance, the flip bifurcation curves F1 and F1-1 give rise to periodic solutions of period 2T. Such solutions are stable for all M and N belonging to the grey regions in Figure 2. Also, we observe that there are regions in the plane (M, N) where there exist more than one stable solution (see for example the C1 of Figure 2 ($M = 1.57,\ N = 0.61$). Instead, one stable solution of period T coexists with one stable solution of period 2T (see point C2 of Figure 1 ($M = 1.38,\ N = 0.38$).

Moreover, trajectories different from T- or 2T-periodic solutions exist. Indeed, the Neimark-Sacker bifurcations (see N-S curves in Figures 1 and 2) give rise to torus stable attractors.

It is also expected that chaotic attractors are very likely to exist in the considered system. For example, two different cascades of flip bifurcations are observed in the regions A and B. The second cascade of flip bifurcations arises by decreasing N, starting from the curve F1-1, through the curve F2-1, and so on.

Summing up, it is clear that the parameterized system offers a wide range of different behaviours as a function of M and N. This analysis may be particularly useful to select most appropriate parameter estimates in domains of the parameter space where the cost function is flat.

Since $F(\theta)$ generally exhibits several local minima, the choice of the optimization starting point is crucial. The insight given by qualitative analysis is used to choose the "region of interest" in the parameter space, where the dynamics of the system is most likely to lie.

The results of the identification of the nonlinear system are drawn in Figure 3.

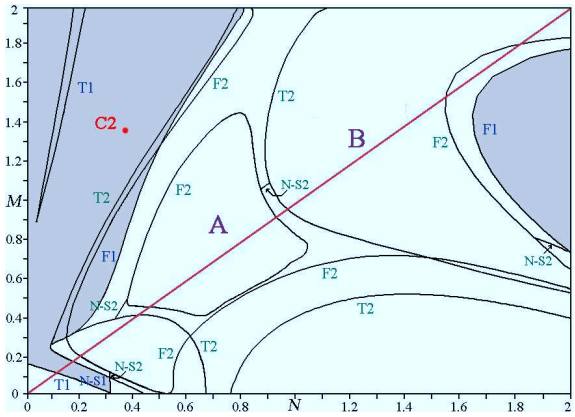

Figure 1: Bifurcation diagram in (M, N) plane. Grey: region of stable T-periodic solutions.

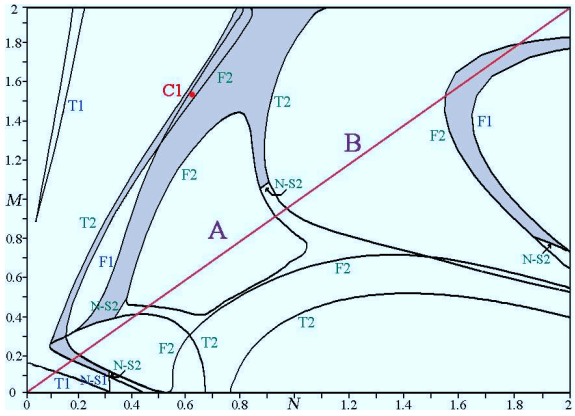

Figure 2: Bifurcation diagram in (M, N) plane. Grey: region of stable 2T-periodic solutions.

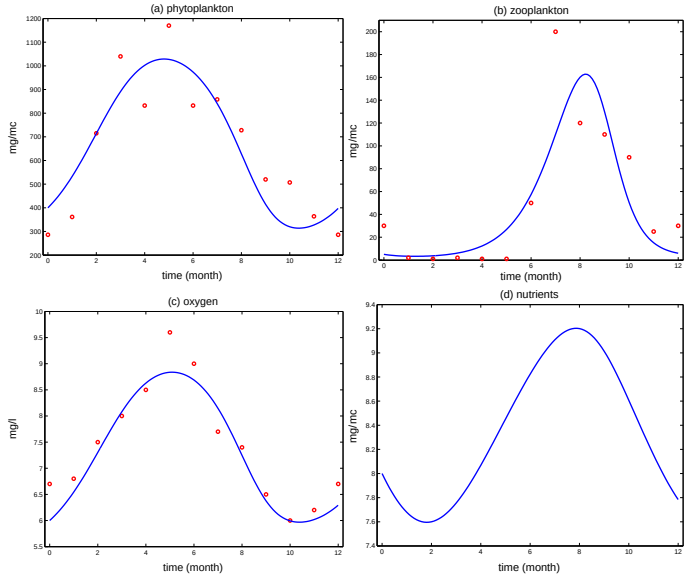

Figure 3: Identification results obtained by the application of the method based on qualitative analysis: model prediction (solid line) and data (∘).

4 Decision Support Systems

Decision Support Systems (DSS), i.e. information systems that help decision making, represent a suitable framework for the implementation of multicriteria approaches. Recently, they have been successfully applied to the sustainable management of coastal basins and water resources. Some applications developed and tested on Mediterranean sites can be found, e.g., in [23, 22, 21, 8].

This paper proposes the general structure of a model-based DSS that is applicable in multiobjective decision problems where several mathematical and analytical models of a (complex) system are available. The DSS structure includes a mechanism for generating the alternatives to be compared. The available models are used to simulate the system under each alternative, and to provide system performance indicators related to each decision criterion. The Analytic Hierarchy Process (AHP) [26] is finally used to evaluate and rank the alternatives on the basis of both the values of the indicators and the interaction with the decision maker. The AHP is a tool for decision making that is suitable in situations where multiple and conflicting criteria are present. However, other multicriteria analysis tools [9], such as reference point [33] or ELECTRE [25] methods, can be used.

Decision support systems cover a wide variety of systems, tools and technologies for informing and supporting decision makers. The concept of a decision support system is in fact extremely broad, and there exists no unique definition of it. If, on the one hand, operation researchers often focus on optimization and simulation models as the "real" DSS, a few common features can be found in the definitions given by several authors. In [27, 29, 24] decision support systems are introduced as interactive computer-based information systems, especially developed for supporting the solution of unstructured management problems by helping managers to retrieve, summarize and analyze decision relevant data, and providing an easy-to-use interface. A DSS includes tools for the design of alternatives, and mechanisms for their comparative analysis, ranking, and selection on the basis of the criteria, objectives, and constraints provided by the users.

Based on the tool or component that provides the dominant functionality in a DSS, Power [24] proposes a taxonomy of decision support systems by distinguishing communication-based, data-based, document-based, knowledge-based, and *model-based* DSS. The last category includes systems integrating different kinds of mathematical and analytical models. Model-based decision support systems exploit the full resolution and detail of simulation models, thus avoiding the pitfalls and limitations of the approximations often used for optimization. Key issues when designing a model-based DSS are the choice of appropriate models and software, and the definition of data formats. Very large databases are usually not needed for model-based decision support systems.

In the real world, decisions are often the result of complex and involved decision making processes characterized by multiple actors and conflicting objectives and interests. The first fundamental step when developing a successful DSS is therefore the decision problem statement, together with a detailed understanding of the actual decision making processes, the actors and the stakeholders (and thus the intended audience and users), their responsibilities, interests, legal and regulatory framework, as well as the institutional structure and the political and socio-economic framework within which the decisions are made. This step is not addressed in this report, and the interested reader is referred to, e.g., [24] and references therein. We assume that the decision problem is structured and presented in terms of:

- The *control options*, i.e. the alternative actions, strategies, and/or policies that can be undertaken to affect the system[1] behavior.
- The *criteria* on which the system performance led by each control option is evaluated.
- The *objectives*, i.e. the type of optimization to be performed on each criterion.
- The *constraints*, establishing bounds for some/all the criteria in order to make the evaluated alternative acceptable or feasible.

The aim of the DSS is to support the choice of a control option that is both effective (i.e. meets the constraints) and efficient (i.e. optimizes the objectives). In this respect, the control option definition and

[1] We distinguish the *system*, i.e. the part of the real world (environment, people, activities, etc.) that is the object of the decision maker's interests and actions, and the *decision support system*, i.e. the tool for supporting the decisions.

design is of central importance. The control options are described by value assignments of the *controllable variables*. On the other hand, the *uncontrollable variables* describe external factors that are not subject to choice, but do affect the system performance. Their role in the DSS can be only viewed in terms of sensitivity and robustness of the final decision.

The criteria are expressed by means of *indicators*, and are used to describe the system and evaluate its behavior and performance under alternative control options. The objectives correspond to indicators whose value has to be either maximized or minimized. The constraints impose a maximum and/or minimum value to the indicators. They may correspond to thresholds defined on the basis of regulations and/or experience, and allow to discard unacceptable alternatives. Additional variables that do not themselves correspond to criteria, but the decision maker might want to constrain, are referred to as *internal variables*. Note that also the controllable and uncontrollable variables could be bounded in order to reflect both physical and practical constraints.

With the above definitions, the basic DSS logic is simple. A set of alternative control options for the system are generated by changing the values of the controllable variables. Each control option leads to a corresponding system performance, which is expressed by indicators. This performance is analyzed, evaluated and compared to arrive at a final preference ranking of the alternatives, and an eventual choice of a preferred alternative as the solution of the decision process. Note that multiple and conflicting criteria require the introduction of direct or indirect *weights* to compare them consistently. The Analytic Hierarchy Process (AHP) provides a method for weighting the criteria based on the decision maker's pairwise comparisons.

4.1 The proposed DSS architecture

Decision making means selecting among alternatives. The function of a DSS is therefore to design and generate alternatives, and to provide the tools for their selection, given the decision maker's criteria, objectives, and constraints. In model-based decision support systems, mathematical and analytical models provide the main functionality by making it possible to simulate the system behavior under different future conditions. The scheme of the proposed model-based DSS architecture is shown in Fig. 4, where the models play a key role between the control option generation and the performance evaluation and comparison (multicriteria analysis). The different component blocks of the DSS architecture will be hereafter described in detail. A first version of the above scheme was presented in [2], where particular emphasis was devoted to an optimization model for resource allocation in coastal lagoon areas characterized by shellfish farming and agriculture.

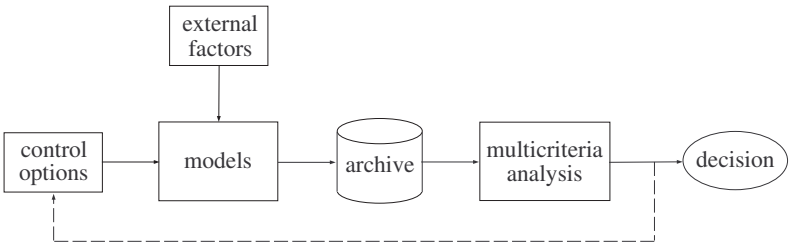

Figure 4: Scheme of the proposed model-based DSS architecture.

4.1.1 Control options

This block provides the alternative (pre-existing or generated on demand) control options by assigning different values to the controllable variables. A discrete approach is assumed, where a finite (possibly very large) set of alternatives is considered. In the following, the number of evaluated alternatives will be denoted by n. The generation mechanism is not specified, since it may depend on the application. Note that the discrete approach does not guarantee optimality, so that the smaller the set to choose from, the less likely it will contain a good (in some sense optimal) solution. On the other hand, for highly complex, non-differentiable systems it may be the only possible approach, which implies that one should

always attempt to generate the largest possible number of alternatives. It is stressed that the generation mechanism can exploit the *feedback* from the multicriteria analysis stage in order to extend or adapt the set of possible solutions in response to the user's preferences.

4.1.2 External factors

This block provides the uncontrollable variables describing the external factors that cannot be controlled/decided by the decision maker, but are needed for the system simulation, and affect its performance (e.g., the weather conditions, the water inflow, etc., for the lagoon models; the prices and the market data for the economic models).

4.1.3 Models

This block represents a suitable interconnection of the models used to describe the system behavior. The use of models is twofold:

- To make simulations and predictions of, e.g., the physical, chemical and biological, as well as the economic and social variables of the system.
- To compute the performance indicators for a quantitative assessment of the evaluated control option.

When the ith control option is considered, $i = 1, ..., n$, the block *Models* receives as inputs both the controllable variables characterizing that particular control option, and the uncontrollable variables. The block output is an m-dimensional vector $I^{(i)}$ (where m is the number of criteria) containing the values of the system performance indicators under the ith control option. Possible constraints imposed on the indicators, as well as on the internal variables, are checked during the simulation. If one or more constraints are violated, the considered alternative is discarded as infeasible.

The internal structure of the block *Models* is specific of the application, being primarily dependent on the type of available models.

For a given control option, the values of the performance indicators are clearly affected by the uncontrollable variables. Hence, in order to perform a fair evaluation of different alternatives, the system performance must be compared under the same external conditions. In addition, in order to make the DSS more robust with respect to varying external conditions, it is strongly recommended to evaluate and compare the alternative options for several value assignments of the uncontrollable variables, and then to consider an average ranking.

4.1.4 Archive

For fixed external conditions, the values of the performance indicators corresponding to the n evaluated control options are stored in the $n \times m$ matrix

$$I = [\ I^{(1)}\ \ldots\ I^{(n)}\ ]^T. \tag{12}$$

4.1.5 Multicriteria analysis

If only the jth criterion is considered ($j = 1, ..., m$), the best control option can be simply selected by taking the optimum over the jth column of the matrix I. However, when all the m criteria are considered, it might likely happen that the optimum over each column is not achieved by the same control option. In this case the selection of the best alternative could be neither direct nor intuitive. This justifies the need for multicriteria analysis tools.

Analytic Hierarchy Process (AHP) is an effective tool to make decisions in situations where multiple and conflicting criteria are present, and both qualitative and quantitative aspects of a decision need to be considered. The AHP provides a ranking of the evaluated alternatives by means of simple matrix calculations and pairwise comparisons. A short description of the AHP is provided in the following.

The *Analytic Hierarchy Process* [26] is a multicriteria decision technique that helps the decision maker to set the priorities and make the best decision by reducing complex evaluations to a series of pairwise comparisons.

The AHP considers a set of evaluation criteria and a set of alternative options. First, a weight is associated to each criterion on the basis of the decision maker's pairwise comparisons of the criteria. Next, a score is associated to the alternative options according to each criterion on the basis of the decision maker's pairwise comparisons of the alternatives. Finally, the AHP combines the criteria weights and the control options scores, thus determining a global score for each control option, and a consequent ranking.

The computations made by the AHP are always guided by the decision maker's experience, and the AHP can thus be considered as a tool that is able to translate the evaluations (both qualitative and quantitative) made by the decision maker into a multicriteria ranking.

A detailed description of the DSS and the application to a coastal lagoon is reported in [2].

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