Sparse Signal Recovery with Random Matrices

Tino Ullrich

tino.ullrich@hcm.uni-bonn.de

Hausdorff Center for Mathematics, University of Bonn

In recent years sparsity has become an important concept in applied mathematics, especially in mathematical signal and image processing, in the numerical treatment of PDEs as well as inverse problems, and statistics. The key idea is that many types of functions and signals arising naturally in these contexts can be described by only a small number of significant degrees of freedom. The novel theory of Compressive Sensing heavily uses this model and predicts, quite surprisingly, that sparse high-dimensional signals can be recovered efficiently from what was previously considered highly incomplete measurements. This discovery has led to a fundamentally new approach towards certain signal and image recovery problems.
Interpreting the signal as a vector x of length N with small support set (non-zero entries) of size s we are able to recover this signal by asking a number m of questions which scales linearly in the sparsity s. To be more precise, these questions are realized by a matrix vector multiplication with a proper matrix A and $y = Ax$. Note, that the system $y = Ax$ is highly under-determined. However, the signal x is recovered from y via a convex L1-norm-minimization program. The exact recovery is guaranteed if the measurement matrix A satisfies the Restricted Isometry Property (RIP). Remarkably, mainly random constructions for measurement matrices A reduce the effort (number of rows m) significantly. Using independent mean-zero sub-Gaussian entries one can prove that m = slog(N/s) rows suffice. The main tool are concentration inequalities for the size of the entries of A. Further constructions involve partial random circulant matrices or Fourier matrices with random samples and provide a similar behavior.
This talk is intended to give an introduction to compressive sensing mainly focused on the RIP of random matrices. If time permits we will also comment on lower bounds for optimal recovery.