SONE RESULTS ON THE BRÜCK COJECTURE

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Abstract

The Value Distribution Theory of Navanlinna is one of the greatest mathematical discoveries of the twentieth. The theory includes the singular direction theory, normal family theory, uniqueness theory and complex equations theory of meromorphic function. The uniqueness theory of meromorphic function mainly studies conditions under which there exists essentially only one function satisfying these conditions. It is well known that any nonconstant polynomial with leading coefficient 1 is determined by its zeros. But it is not true for the transcendental entire or meromorphic functions. Therefore, how to uniquely determine a meromorphic function is interesting and complex.

In 1996, for the one CM shared value of functions, R.Brück proposed the following famous conjecture: Let $f(z)$ be a nonconstant entire function. Suppose that $\rho_2(f)$ is not a positive integer or infinite. If $f(z)$ and $f'(z)$ share one finite value a CM, then $f'(z) - a = c(f(z) - a)$, where c is some nonzero constant, $\rho_2(f)$ is the hyper-order of $f(z)$.

This conjecture has been a well known unsolved problem although dozens of related papers have been published. Kinds of special forms of the Brück conjecture have been shown, for example, the Brück conjecture concerning Nevanlinna deficiency, small functions, power functions and difference operators, etc. In this talk, we will introduce the research progress of this conjecture.

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