Blind approximation of surfaces

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We consider the problem of discovering the shape of an unknown surface S of $\mathbb{R}^3$ through an adaptive process of probing. The goal is to find a strategy for the sequence of probes that guarantees precise approximation of S after a minimal number of probes. Differently from most work in surface reconstruction, the data are not given prior to the reconstruction phase but must be computed interactively, the new probe depending on the outcomes of the previous probes.

The problem is motivated by applications in geometric modeling, function approximation and robotics [4]. There exist several variants of the general problem depending on the probe model, the class of shapes under consideration, and the constraints on the motions of the probing device.

Probing strategies have been proposed for planar convex objects using line probes (returning the first point hit by a translating line) [2] and some other probe models are analyzed by Rote [3]. We consider the case of $C^{1,1}$ surfaces with a known positive lower bound on the reach (also known as the normal injectivity radius) of S. The Delaunay refinement technique for surface mesh generation [1] can be adapted to obtain blind approximations of surfaces. The algorithm probes along line segments (Voronoi edges) and repetedly inserts new points on S until the Delaunay triangulation of the sample points restricted to S has small facets (with respect to the specified error bound ε). Upon termination, we have a triangulated surface $\hat{S}$ that is at Hausdorff distance ε (for any specified ε) from S, and isotopic to S. The number of vertices of $\hat{S}$ is $O(\frac{\text{area}(S)}{\varepsilon})$, which is asymptotically optimal []. We also analyze the combinatorial complexity (i.e. the cost of maintaining the data structures) and the information-based cost (i.e. the number of probes), and show both of them to be $O(\frac{1}{\varepsilon}\log\frac{1}{\varepsilon})$. We will discuss also the cost of moving the probing device.

Experimental results on various types of surfaces will be presented.

Joint work with Steve Oudot (INRIA).

References

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